1.5 Analyzing Graphs of Functions

What you should learn

- Use the Vertical Line Test for functions.
- Find the zeros of functions.
- Determine intervals on which functions are increasing or decreasing and determine relative maximum and relative minimum values of functions.
- Determine the average rate of change of a function.
- Identify even and odd functions.

Why you should learn it

Graphs of functions can help you visualize relationships between variables in real life. For instance, in Exercise 86 on page 64, you will use the graph of a function to represent visually the temperature for a city over a 24-hour period.

The Graph of a Function

In Section 1.4, you studied functions from an algebraic point of view. In this section, you will study functions from a graphical perspective.

The **graph of a function** f is the collection of ordered pairs $(x, f(x))$ such that x is in the domain of f. As you study this section, remember that

$x =$ the directed distance from the y-axis

$y = f(x) =$ the directed distance from the x-axis

as shown in Figure 1.52.

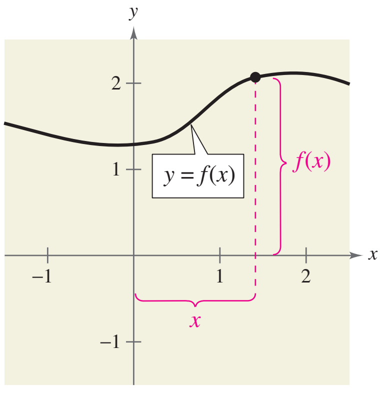


FIGURE 1.52

Example 1 **Finding the Domain and Range of a Function**

Use the graph of the function f, shown in Figure 1.53, to find (a) the domain of f, (b) the function values $f(-1)$ and $f(2)$, and (c) the range of f.

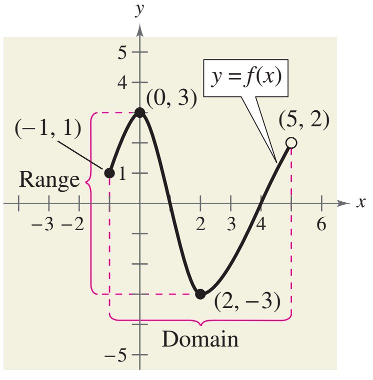


FIGURE 1.53

Solution

a. The closed dot at $(-1, 1)$ indicates that $x = -1$ is in the domain of f, whereas the open dot at $(5, 2)$ indicates that $x = 5$ is not in the domain. So, the domain of f is all x in the interval $[-1, 5)$.

b. Because $(-1, 1)$ is a point on the graph of f, it follows that $f(-1) = 1$. Similarly, because $(2, -3)$ is a point on the graph of f, it follows that $f(2) = -3$.

c. Because the graph does not extend below $f(2) = -3$ or above $f(0) = 3$, the range of f is the interval $[-3, 3]$.

✓CHECKPOINT Now try Exercise 1.

The use of dots (open or closed) at the extreme left and right points of a graph indicates that the graph does not extend beyond these points. If no such dots are shown, assume that the graph extends beyond these points.

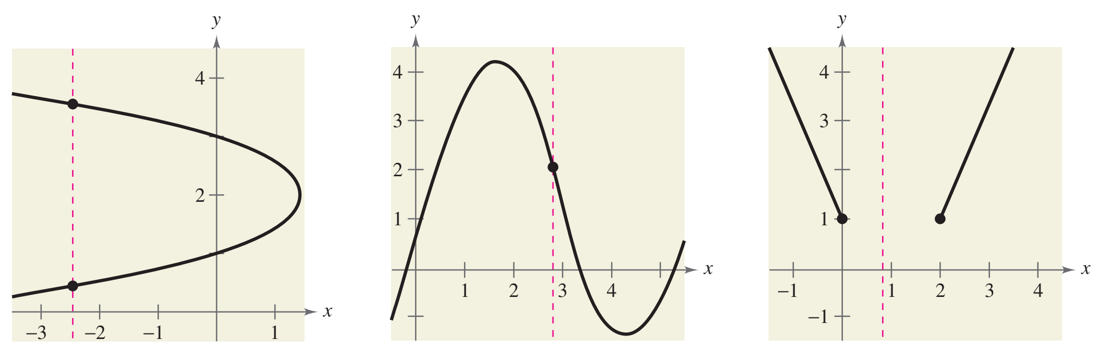
By the definition of a function, at most one y-value corresponds to a given x-value. This means that the graph of a function cannot have two or more different points with the same x-coordinate, and no two points on the graph of a function can be vertically above or below each other. It follows, then, that a vertical line can intersect the graph of a function at most once. This observation provides a convenient visual test called the **Vertical Line Test** for functions.

Vertical Line Test for Functions

A set of points in a coordinate plane is the graph of y as a function of x if and only if no *vertical* line intersects the graph at more than one point.

Example 2 Vertical Line Test for Functions

Use the Vertical Line Test to decide whether the graphs in Figure 1.54 represent y as a function of x.



(a) (b) (c)

FIGURE 1.54

Solution

a. This *is not* a graph of y as a function of x, because you can find a vertical line that intersects the graph twice. That is, for a particular input x, there is more than one output y.

b. This *is* a graph of y as a function of x, because every vertical line intersects the graph at most once. That is, for a particular input x, there is at most one output y.

c. This *is* a graph of y as a function of x. (Note that if a vertical line does not intersect the graph, it simply means that the function is undefined for that particular value of x.) That is, for a particular input x, there is at most one output y.

✓CHECKPOINT Now try Exercise 9.

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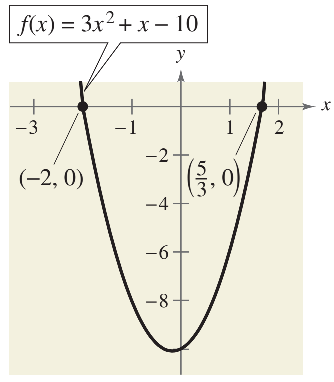
Zeros of a Function

If the graph of a function of x has an x-intercept at $(a, 0)$, then a is a **zero** of the function.

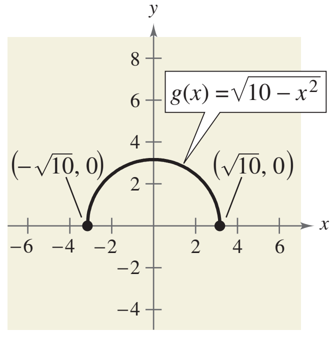
Zeros of a Function

The **zeros of a function** f of x are the x-values for which $f(x) = 0$.



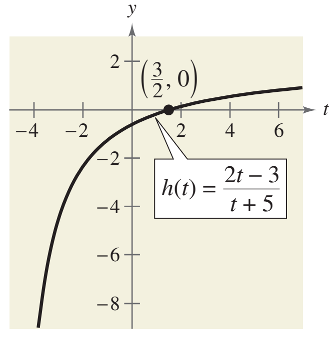
Zeros of f: $x = -2, x = \frac{5}{3}$

FIGURE 1.55



Zeros of g: $x = \pm\sqrt{10}$

FIGURE 1.56



Zero of h: $t = \frac{3}{2}$

FIGURE 1.57

Example 3 **Finding the Zeros of a Function**

Find the zeros of each function.

a. $f(x) = 3x^2 + x - 10$ **b.** $g(x) = \sqrt{10 - x^2}$ **c.** $h(t) = \dfrac{2t - 3}{t + 5}$

Solution

To find the zeros of a function, set the function equal to zero and solve for the independent variable.

a.

$3x^2 + x - 10 = 0$ Set $f(x)$ equal to 0.

$(3x - 5)(x + 2) = 0$ Factor.

$3x - 5 = 0 \Rightarrow x = \frac{5}{3}$ Set 1st factor equal to 0.

$x + 2 = 0 \Rightarrow x = -2$ Set 2nd factor equal to 0.

The zeros of f are $x = \frac{5}{3}$ and $x = -2$. In Figure 1.55, note that the graph of f has $\left(\frac{5}{3}, 0\right)$ and $(-2, 0)$ as its x-intercepts.

b.

$\sqrt{10 - x^2} = 0$ Set $g(x)$ equal to 0.

$10 - x^2 = 0$ Square each side.

$10 = x^2$ Add x^2 to each side.

$\pm\sqrt{10} = x$ Extract square roots.

The zeros of g are $x = -\sqrt{10}$ and $x = \sqrt{10}$. In Figure 1.56, note that the graph of g has $(-\sqrt{10}, 0)$ and $(\sqrt{10}, 0)$ as its x-intercepts.

c.

$\dfrac{2t - 3}{t + 5} = 0$ Set $h(t)$ equal to 0.

$2t - 3 = 0$ Multiply each side by $t + 5$.

$2t = 3$ Add 3 to each side.

$t = \dfrac{3}{2}$ Divide each side by 2.

The zero of h is $t = \frac{3}{2}$. In Figure 1.57, note that the graph of h has $\left(\frac{3}{2}, 0\right)$ as its t-intercept.

CHECKPOINT Now try Exercise 15.

Increasing and Decreasing Functions

The more you know about the graph of a function, the more you know about the function itself. Consider the graph shown in Figure 1.58. As you move from *left to right*, this graph falls from $x = -2$ to $x = 0$, is constant from $x = 0$ to $x = 2$, and rises from $x = 2$ to $x = 4$.

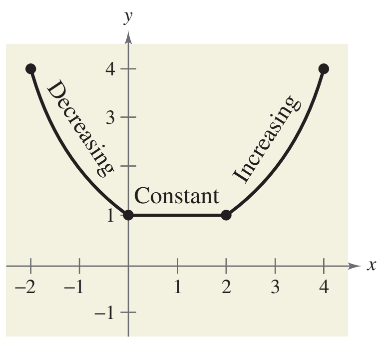


FIGURE 1.58

Increasing, Decreasing, and Constant Functions

A function f is **increasing** on an interval if, for any x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

A function f is **decreasing** on an interval if, for any x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.

A function f is **constant** on an interval if, for any x_1 and x_2 in the interval, $f(x_1) = f(x_2)$.

Example 4 Increasing and Decreasing Functions

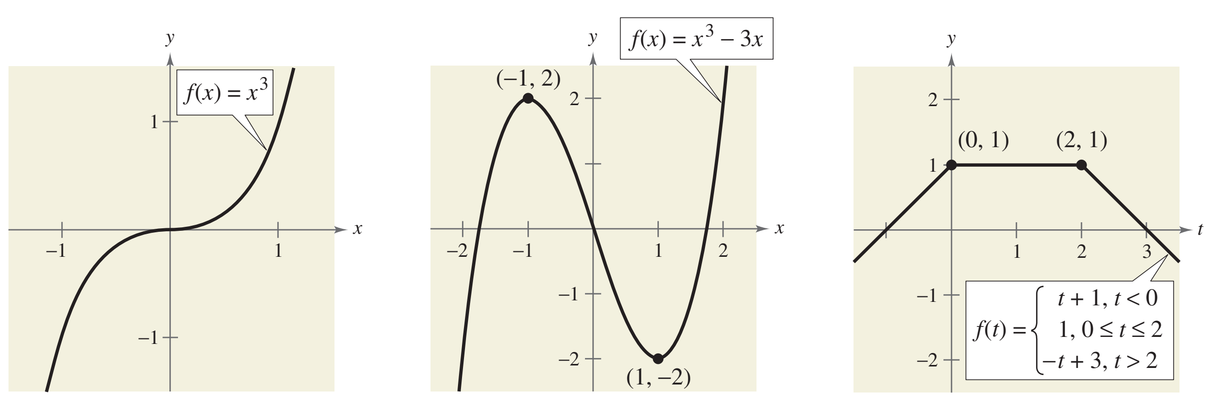
Use the graphs in Figure 1.59 to describe the increasing or decreasing behavior of each function.

Solution

a. This function is increasing over the entire real line.

b. This function is increasing on the interval $(-\infty, -1)$, decreasing on the interval $(-1, 1)$, and increasing on the interval $(1, \infty)$.

c. This function is increasing on the interval $(-\infty, 0)$, constant on the interval $(0, 2)$, and decreasing on the interval $(2, \infty)$.



(a) (b) (c)

FIGURE 1.59

CHECKPOINT Now try Exercise 33.

To help you decide whether a function is increasing, decreasing, or constant on an interval, you can evaluate the function for several values of x. However, calculus is needed to determine, for certain, all intervals on which a function is increasing, decreasing, or constant.

STUDY TIP

A relative minimum or relative maximum is also referred to as a *local* minimum or *local* maximum.

The points at which a function changes its increasing, decreasing, or constant behavior are helpful in determining the **relative minimum** or **relative maximum** values of the function.

Definitions of Relative Minimum and Relative Maximum

A function value $f(a)$ is called a **relative minimum** of f if there exists an interval (x_1, x_2) that contains a such that

$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \leq f(x).$$

A function value $f(a)$ is called a **relative maximum** of f if there exists an interval (x_1, x_2) that contains a such that

$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \geq f(x).$$

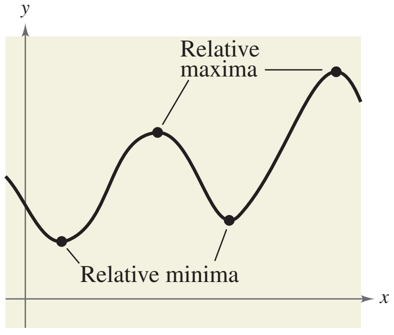


FIGURE 1.60

Figure 1.60 shows several different examples of relative minima and relative maxima. In Section 2.1, you will study a technique for finding the *exact point* at which a second-degree polynomial function has a relative minimum or relative maximum. For the time being, however, you can use a graphing utility to find reasonable approximations of these points.

Example 5 Approximating a Relative Minimum

Use a graphing utility to approximate the relative minimum of the function given by $f(x) = 3x^2 - 4x - 2$.

Solution

The graph of f is shown in Figure 1.61. By using the *zoom* and *trace* features or the *minimum* feature of a graphing utility, you can estimate that the function has a relative minimum at the point

$(0.67, -3.33)$. Relative minimum

Later, in Section 2.1, you will be able to determine that the exact point at which the relative minimum occurs is $\left(\frac{2}{3}, -\frac{10}{3}\right)$.

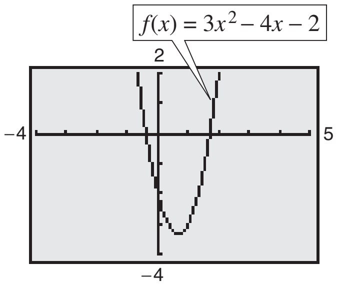


FIGURE 1.61

CHECKPOINT Now try Exercise 49.

You can also use the *table* feature of a graphing utility to approximate numerically the relative minimum of the function in Example 5. Using a table that begins at 0.6 and increments the value of x by 0.01, you can approximate that the minimum of $f(x) = 3x^2 - 4x - 2$ occurs at the point $(0.67, -3.33)$.

Technology

If you use a graphing utility to estimate the x- and y-values of a relative minimum or relative maximum, the *zoom* feature will often produce graphs that are nearly flat. To overcome this problem, you can manually change the vertical setting of the viewing window. The graph will stretch vertically if the values of Ymin and Ymax are closer together.

Average Rate of Change

In Section 1.3, you learned that the slope of a line can be interpreted as a *rate of change*. For a nonlinear graph whose slope changes at each point, the **average rate of change** between any two points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ is the slope of the line through the two points (see Figure 1.62). The line through the two points is called the **secant line,** and the slope of this line is denoted as m_{sec}.

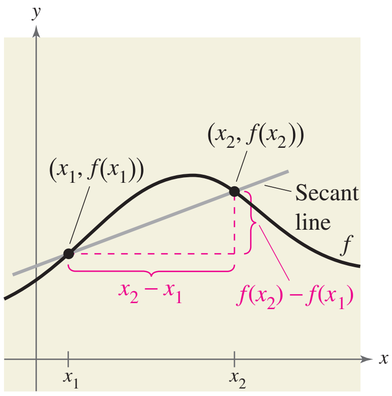


FIGURE 1.62

$$\begin{aligned}\text{Average rate of change of } f \text{ from } x_1 \text{ to } x_2 &= \frac{f(x_2) - f(x_1)}{x_2 - x_1} \\ &= \frac{\text{change in } y}{\text{change in } x} \\ &= m_{sec}\end{aligned}$$

Example 6 Average Rate of Change of a Function

Find the average rates of change of $f(x) = x^3 - 3x$ (a) from $x_1 = -2$ to $x_2 = 0$ and (b) from $x_1 = 0$ to $x_2 = 1$ (see Figure 1.63).

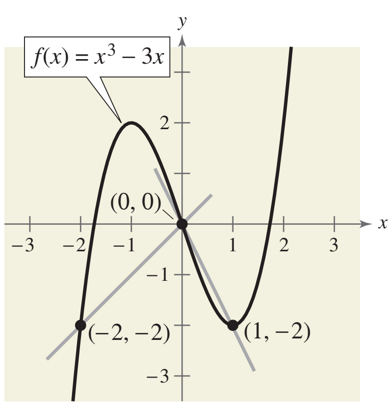


FIGURE 1.63

Solution

a. The average rate of change of f from $x_1 = -2$ to $x_2 = 0$ is

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(0) - f(-2)}{0 - (-2)} = \frac{0 - (-2)}{2} = 1.$$

Secant line has positive slope.

b. The average rate of change of f from $x_1 = 0$ to $x_2 = 1$ is

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1) - f(0)}{1 - 0} = \frac{-2 - 0}{1} = -2.$$

Secant line has negative slope.

CHECKPOINT Now try Exercise 63.

Example 7 Finding Average Speed

The distance s (in feet) a moving car is from a stoplight is given by the function $s(t) = 20t^{3/2}$, where t is the time (in seconds). Find the average speed of the car (a) from $t_1 = 0$ to $t_2 = 4$ seconds and (b) from $t_1 = 4$ to $t_2 = 9$ seconds.

Solution

a. The average speed of the car from $t_1 = 0$ to $t_2 = 4$ seconds is

$$\frac{s(t_2) - s(t_1)}{t_2 - t_1} = \frac{s(4) - s(0)}{4 - (0)} = \frac{160 - 0}{4} = 40 \text{ feet per second.}$$

b. The average speed of the car from $t_1 = 4$ to $t_2 = 9$ seconds is

$$\frac{s(t_2) - s(t_1)}{t_2 - t_1} = \frac{s(9) - s(4)}{9 - 4} = \frac{540 - 160}{5} = 76 \text{ feet per second.}$$

CHECKPOINT Now try Exercise 89.

Exploration

Use the information in Example 7 to find the average speed of the car from $t_1 = 0$ to $t_2 = 9$ seconds. Explain why the result is less than the value obtained in part (b).

Even and Odd Functions

In Section 1.2, you studied different types of symmetry of a graph. In the terminology of functions, a function is said to be **even** if its graph is symmetric with respect to the y-axis and to be **odd** if its graph is symmetric with respect to the origin. The symmetry tests in Section 1.2 yield the following tests for even and odd functions.

Tests for Even and Odd Functions

A function $y = f(x)$ is **even** if, for each x in the domain of f,

$$f(-x) = f(x).$$

A function $y = f(x)$ is **odd** if, for each x in the domain of f,

$$f(-x) = -f(x).$$

Exploration

Graph each of the functions with a graphing utility. Determine whether the function is *even*, *odd*, or *neither*.

$$f(x) = x^2 - x^4$$

$$g(x) = 2x^3 + 1$$

$$h(x) = x^5 - 2x^3 + x$$

$$j(x) = 2 - x^6 - x^8$$

$$k(x) = x^5 - 2x^4 + x - 2$$

$$p(x) = x^9 + 3x^5 - x^3 + x$$

What do you notice about the equations of functions that are odd? What do you notice about the equations of functions that are even? Can you describe a way to identify a function as odd or even by inspecting the equation? Can you describe a way to identify a function as neither odd nor even by inspecting the equation?

Example 8 Even and Odd Functions

a. The function $g(x) = x^3 - x$ is odd because $g(-x) = -g(x)$, as follows.

$g(-x) = (-x)^3 - (-x)$ — Substitute $-x$ for x.

$= -x^3 + x$ — Simplify.

$= -(x^3 - x)$ — Distributive Property

$= -g(x)$ — Test for odd function

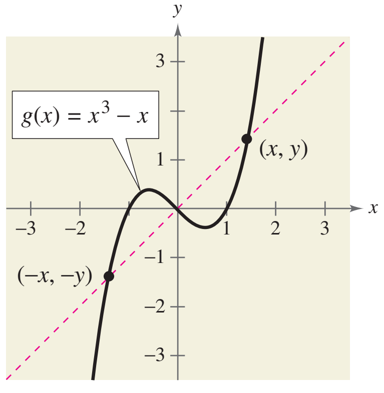
b. The function $h(x) = x^2 + 1$ is even because $h(-x) = h(x)$, as follows.

$h(-x) = (-x)^2 + 1$ — Substitute $-x$ for x.

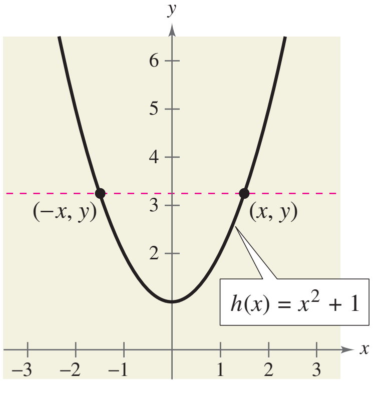
$= x^2 + 1$ — Simplify.

$= h(x)$ — Test for even function

The graphs and symmetry of these two functions are shown in Figure 1.64.



(a) Symmetric to origin: Odd Function



(b) Symmetric to y-axis: Even Function

FIGURE 1.64

CHECKPOINT Now try Exercise 71.

Additional Example

Is the function given by $f(x) = x^3 - 1$ even, odd, or neither?

Solution

Substituting $-x$ for x,

$f(-x) = (-x)^3 - 1 = -x^3 - 1$.

Because $f(x) = x^3 - 1$ and $f(-x) = -x^3 - 1$, $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$. The function is neither even nor odd.

1.5 Exercises

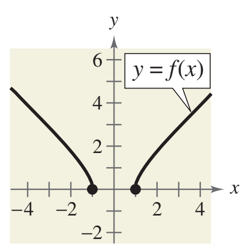
VOCABULARY CHECK: Fill in the blanks.

1. The graph of a function f is the collection of ________ ________ or $(x, f(x))$ such that x is in the domain of f.
2. The ________ ________ ________ is used to determine whether the graph of an equation is a function of y in terms of x.
3. The ________ of a function f are the values of x for which $f(x) = 0$.
4. A function f is ________ on an interval if, for any x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.
5. A function value $f(a)$ is a relative ________ of f if there exists an interval (x_1, x_2) containing a such that $x_1 < x < x_2$ implies $f(a) \geq f(x)$.
6. The ________ ________ ________ ________ between any two points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ is the slope of the line through the two points, and this line is called the ________ line.
7. A function f is ________ if for the each x in the domain of f, $f(-x) = -f(x)$.
8. A function f is ________ if its graph is symmetric with respect to the y-axis.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–4, use the graph of the function to find the domain and range of *f*.

1.



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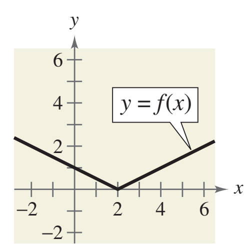

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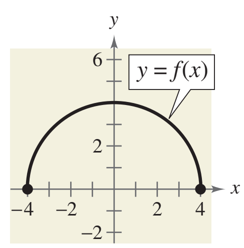

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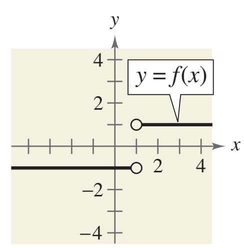
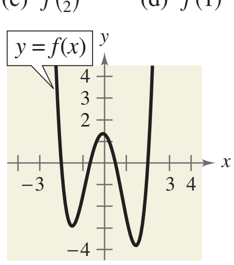
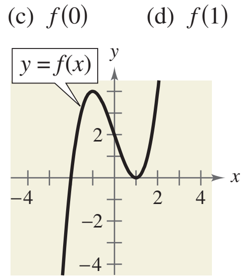


In Exercises 5–8, use the graph of the function to find the indicated function values.

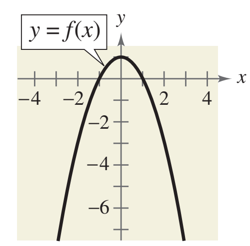
5. (a) $f(-2)$ (b) $f(-1)$ (c) $f\left(\frac{1}{2}\right)$ (d) $f(1)$



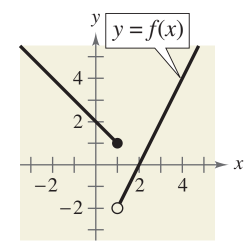
6. (a) $f(-1)$ (b) $f(2)$ (c) $f(0)$ (d) $f(1)$



7. (a) $f(-2)$ (b) $f(1)$ (c) $f(0)$ (d) $f(2)$

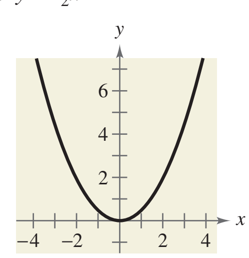


8. (a) $f(2)$ (b) $f(1)$ (c) $f(3)$ (d) $f(-1)$

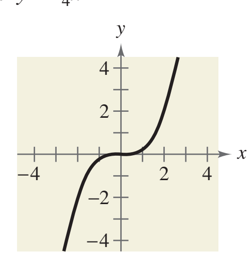


In Exercises 9–14, use the Vertical Line Test to determine whether *y* is a function of *x*. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

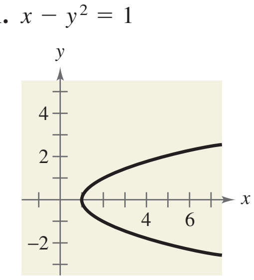
9. $y = \frac{1}{2}x^2$



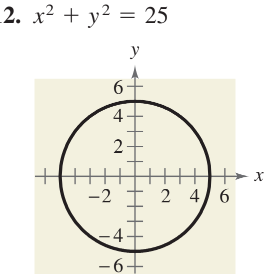
10. $y = \frac{1}{4}x^3$



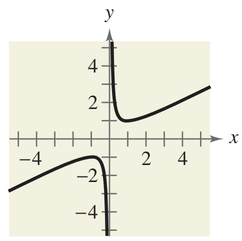
11. $x - y^2 = 1$



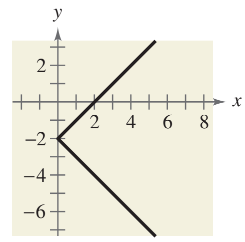
12. $x^2 + y^2 = 25$



13. $x^2 = 2xy - 1$



14. $x = |y + 2|$



In Exercises 15–24, find the zeros of the function algebraically.

15. $f(x) = 2x^2 - 7x - 30$

16. $f(x) = 3x^2 + 22x - 16$

17. $f(x) = \dfrac{x}{9x^2 - 4}$

18. $f(x) = \dfrac{x^2 - 9x + 14}{4x}$

19. $f(x) = \frac{1}{2}x^3 - x$

20. $f(x) = x^3 - 4x^2 - 9x + 36$

21. $f(x) = 4x^3 - 24x^2 - x + 6$

22. $f(x) = 9x^4 - 25x^2$

23. $f(x) = \sqrt{2x} - 1$

24. $f(x) = \sqrt{3x + 2}$

In Exercises 25–30, (a) use a graphing utility to graph the function and find the zeros of the function and (b) verify your results from part (a) algebraically.

25. $f(x) = 3 + \dfrac{5}{x}$

26. $f(x) = x(x - 7)$

27. $f(x) = \sqrt{2x + 11}$

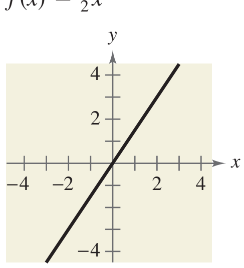
28. $f(x) = \sqrt{3x - 14} - 8$

29. $f(x) = \dfrac{3x - 1}{x - 6}$

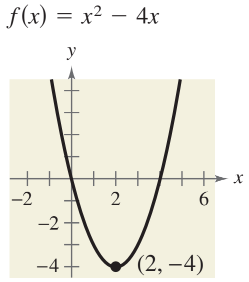
30. $f(x) = \dfrac{2x^2 - 9}{3 - x}$

In Exercises 31–38, determine the intervals over which the function is increasing, decreasing, or constant.

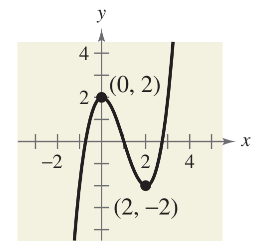
31. $f(x) = \frac{3}{2}x$



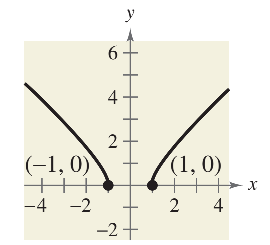
32. $f(x) = x^2 - 4x$



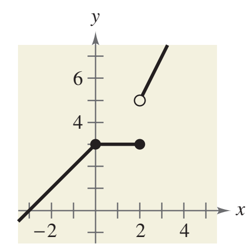
33. $f(x) = x^3 - 3x^2 + 2$



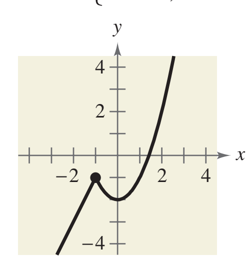
34. $f(x) = \sqrt{x^2 - 1}$



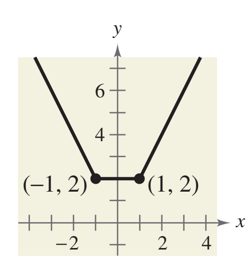
35. $f(x) = \begin{cases} x + 3, & x \le 0 \\ 3, & 0 < x \le 2 \\ 2x + 1, & x > 2 \end{cases}$



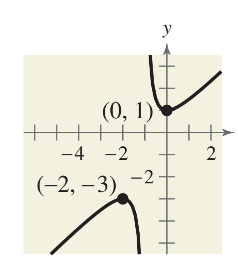
36. $f(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 - 2, & x > -1 \end{cases}$



37. $f(x) = |x + 1| + |x - 1|$



38. $f(x) = \dfrac{x^2 + x + 1}{x + 1}$



In Exercises 39–48, (a) use a graphing utility to graph the function and visually determine the intervals over which the function is increasing, decreasing, or constant, and (b) make a table of values to verify whether the function is increasing, decreasing, or constant over the intervals you identified in part (a).

39. $f(x) = 3$ **40.** $g(x) = x$

41. $g(s) = \dfrac{s^2}{4}$ **42.** $h(x) = x^2 - 4$

43. $f(t) = -t^4$ **44.** $f(x) = 3x^4 - 6x^2$

45. $f(x) = \sqrt{1-x}$ **46.** $f(x) = x\sqrt{x+3}$

47. $f(x) = x^{3/2}$ **48.** $f(x) = x^{2/3}$

In Exercises 49–54, use a graphing utility to graph the function and approximate (to two decimal places) any relative minimum or relative maximum values.

49. $f(x) = (x-4)(x+2)$ **50.** $f(x) = 3x^2 - 2x - 5$

51. $f(x) = -x^2 + 3x - 2$ **52.** $f(x) = -2x^2 + 9x$

53. $f(x) = x(x-2)(x+3)$

54. $f(x) = x^3 - 3x^2 - x + 1$

In Exercises 55–62, graph the function and determine the interval(s) for which $f(x) \geq 0$.

55. $f(x) = 4 - x$ **56.** $f(x) = 4x + 2$

57. $f(x) = x^2 + x$ **58.** $f(x) = x^2 - 4x$

59. $f(x) = \sqrt{x-1}$ **60.** $f(x) = \sqrt{x+2}$

61. $f(x) = -(1 + |x|)$ **62.** $f(x) = \frac{1}{2}(2 + |x|)$

In Exercises 63–70, find the average rate of change of the function from x_1 to x_2.

	Function	*x-Values*
63.	$f(x) = -2x + 15$	$x_1 = 0, x_2 = 3$
64.	$f(x) = 3x + 8$	$x_1 = 0, x_2 = 3$
65.	$f(x) = x^2 + 12x - 4$	$x_1 = 1, x_2 = 5$
66.	$f(x) = x^2 - 2x + 8$	$x_1 = 1, x_2 = 5$
67.	$f(x) = x^3 - 3x^2 - x$	$x_1 = 1, x_2 = 3$
68.	$f(x) = -x^3 + 6x^2 + x$	$x_1 = 1, x_2 = 6$
69.	$f(x) = -\sqrt{x-2} + 5$	$x_1 = 3, x_2 = 11$
70.	$f(x) = -\sqrt{x+1} + 3$	$x_1 = 3, x_2 = 8$

In Exercises 71–76, determine whether the function is even, odd, or neither. Then describe the symmetry.

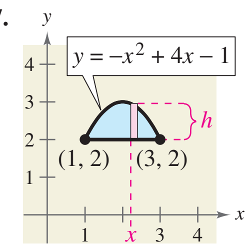
71. $f(x) = x^6 - 2x^2 + 3$ **72.** $h(x) = x^3 - 5$

73. $g(x) = x^3 - 5x$ **74.** $f(x) = x\sqrt{1-x^2}$

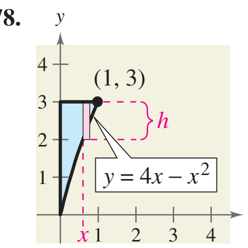
75. $f(t) = t^2 + 2t - 3$ **76.** $g(s) = 4s^{2/3}$

In Exercises 77–80, write the height h of the rectangle as a function of x.

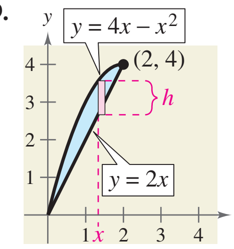
77.



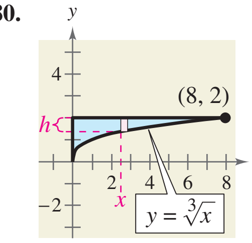
78.



79.

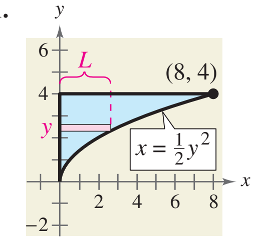


80.

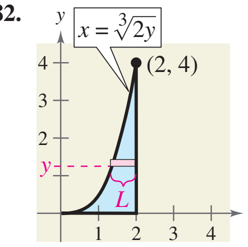


In Exercises 81–84, write the length L of the rectangle as a function of y.

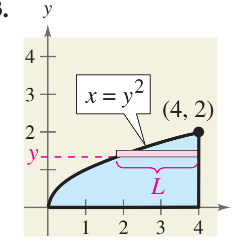
81.



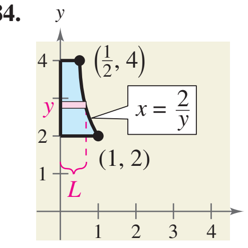
82.



83.



84.



85. ***Electronics*** The number of lumens (time rate of flow of light) L from a fluorescent lamp can be approximated by the model

$$L = -0.294x^2 + 97.744x - 664.875, \quad 20 \leq x \leq 90$$

where x is the wattage of the lamp.

(a) Use a graphing utility to graph the function.

(b) Use the graph from part (a) to estimate the wattage necessary to obtain 2000 lumens.

Model It

86. ***Data Analysis: Temperature*** The table shows the temperature y (in degrees Fahrenheit) of a certain city over a 24-hour period. Let x represent the time of day, where $x = 0$ corresponds to 6 A.M.

Time, x	Temperature, y
0	34
2	50
4	60
6	64
8	63
10	59
12	53
14	46
16	40
18	36
20	34
22	37
24	45

A model that represents these data is given by

$$y = 0.026x^3 - 1.03x^2 + 10.2x + 34, \quad 0 \le x \le 24.$$

(a) Use a graphing utility to create a scatter plot of the data. Then graph the model in the same viewing window.

(b) How well does the model fit the data?

(c) Use the graph to approximate the times when the temperature was increasing and decreasing.

(d) Use the graph to approximate the maximum and minimum temperatures during this 24-hour period.

(e) Could this model be used to predict the temperature for the city during the next 24-hour period? Why or why not?

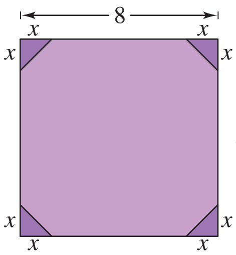
87. ***Coordinate Axis Scale*** Each function models the specified data for the years 1995 through 2005, with $t = 5$ corresponding to 1995. Estimate a reasonable scale for the vertical axis (e.g., hundreds, thousands, millions, etc.) of the graph and justify your answer. (There are many correct answers.)

(a) $f(t)$ represents the average salary of college professors.

(b) $f(t)$ represents the U.S. population.

(c) $f(t)$ represents the percent of the civilian work force that is unemployed.

88. ***Geometry*** Corners of equal size are cut from a square with sides of length 8 meters (see figure).



(a) Write the area A of the resulting figure as a function of x. Determine the domain of the function.

(b) Use a graphing utility to graph the area function over its domain. Use the graph to find the range of the function.

(c) Identify the figure that would result if x were chosen to be the maximum value in the domain of the function. What would be the length of each side of the figure?

89. ***Digital Music Sales*** The estimated revenues r (in billions of dollars) from sales of digital music from 2002 to 2007 can be approximated by the model

$$r = 15.639t^3 - 104.75t^2 + 303.5t - 301, \quad 2 \le t \le 7$$

where t represents the year, with $t = 2$ corresponding to 2002. (Source: *Fortune*)

(a) Use a graphing utility to graph the model.

(b) Find the average rate of change of the model from 2002 to 2007. Interpret your answer in the context of the problem.

90. ***Foreign College Students*** The numbers of foreign students F (in thousands) enrolled in colleges in the United States from 1992 to 2002 can be approximated by the model.

$$F = 0.004t^4 + 0.46t^2 + 431.6, \quad 2 \le t \le 12$$

where t represents the year, with $t = 2$ corresponding to 1992. (Source: Institute of International Education)

(a) Use a graphing utility to graph the model.

(b) Find the average rate of change of the model from 1992 to 2002. Interpret your answer in the context of the problem.

(c) Find the five-year time periods when the rate of change was the greatest and the least.

Physics **In Exercises 91–96, (a) use the position equation $s = -16t^2 + v_0t + s_0$ to write a function that represents the situation, (b) use a graphing utility to graph the function, (c) find the average rate of change of the function from t_1 to t_2, (d) interpret your answer to part (c) in the context of the problem, (e) find the equation of the secant line through t_1 and t_2, and (f) graph the secant line in the same viewing window as your position function.**

91. An object is thrown upward from a height of 6 feet at a velocity of 64 feet per second.

$t_1 = 0, t_2 = 3$

92. An object is thrown upward from a height of 6.5 feet at a velocity of 72 feet per second.

$t_1 = 0, t_2 = 4$

93. An object is thrown upward from ground level at a velocity of 120 feet per second.

$t_1 = 3, t_2 = 5$

94. An object is thrown upward from ground level at a velocity of 96 feet per second.

$t_1 = 2, t_2 = 5$

95. An object is dropped from a height of 120 feet.

$t_1 = 0, t_2 = 2$

96. An object is dropped from a height of 80 feet.

$t_1 = 1, t_2 = 2$

Synthesis

True or False? **In Exercises 97 and 98, determine whether the statement is true or false. Justify your answer.**

97. A function with a square root cannot have a domain that is the set of real numbers.

98. It is possible for an odd function to have the interval $[0, \infty)$ as its domain.

99. If f is an even function, determine whether g is even, odd, or neither. Explain.

(a) $g(x) = -f(x)$

(b) $g(x) = f(-x)$

(c) $g(x) = f(x) - 2$

(d) $g(x) = f(x - 2)$

100. ***Think About It*** Does the graph in Exercise 11 represent x as a function of y? Explain.

Think About It **In Exercises 101–104, find the coordinates of a second point on the graph of a function f if the given point is on the graph and the function is (a) even and (b) odd.**

101. $\left(-\frac{3}{2}, 4\right)$

102. $\left(-\frac{5}{3}, -7\right)$

103. $(4, 9)$

104. $(5, -1)$

105. ***Writing*** Use a graphing utility to graph each function. Write a paragraph describing any similarities and differences you observe among the graphs.

(a) $y = x$ (b) $y = x^2$

(c) $y = x^3$ (d) $y = x^4$

(e) $y = x^5$ (f) $y = x^6$

106. ***Conjecture*** Use the results of Exercise 105 to make a conjecture about the graphs of $y = x^7$ and $y = x^8$. Use a graphing utility to graph the functions and compare the results with your conjecture.

Skills Review

In Exercises 107–110, solve the equation.

107. $x^2 - 10x = 0$

108. $100 - (x - 5)^2 = 0$

109. $x^3 - x = 0$

110. $16x^2 - 40x + 25 = 0$

In Exercises 111–114, evaluate the function at each specified value of the independent variable and simplify.

111. $f(x) = 5x - 8$

(a) $f(9)$ (b) $f(-4)$ (c) $f(x - 7)$

112. $f(x) = x^2 - 10x$

(a) $f(4)$ (b) $f(-8)$ (c) $f(x - 4)$

113. $f(x) = \sqrt{x - 12} - 9$

(a) $f(12)$ (b) $f(40)$ (c) $f\left(-\sqrt{36}\right)$

114. $f(x) = x^4 - x - 5$

(a) $f(-1)$ (b) $f\left(\frac{1}{2}\right)$ (c) $f\left(2\sqrt{3}\right)$

In Exercises 115 and 116, find the difference quotient and simplify your answer.

115. $f(x) = x^2 - 2x + 9, \quad \dfrac{f(3 + h) - f(3)}{h}, \quad h \neq 0$

116. $f(x) = 5 + 6x - x^2, \quad \dfrac{f(6 + h) - f(6)}{h}, \quad h \neq 0$