1-20-12

derivative of antiderivative

Calculus - 2nd FTC

Chain rule

Find derivative

① a) $\int_0^x \frac{\sin(t)}{t}\,dt = \boxed{\frac{\sin x}{x}}$

b) $\int_0^x e^{-t^2}\,dt = \boxed{e^{-x^2}}$

c) $\int_1^{\cos x} \left(\frac{1}{t}\right) dt = \frac{1}{\cos x} \cdot -\sin x = \boxed{-\tan x}$

d) $\int_0^1 e^{\tan^2 t}\,dt =$ (No variable) derivative of constant $= \boxed{0}$

e) $\int_x^{x^2} \frac{1}{2t}\,dt,\ x > 0 = \frac{1}{2x^2} \cdot 2x - \frac{1}{2x}(1) = \frac{1}{x} - \frac{1}{2x} = \boxed{\frac{1}{2x}}$

f) $\int_x^2 \cos(t^2)\,dt = -\int_2^x \cos(t^2) = \boxed{-\cos(x^2)}$

g) $\int_1^{\sqrt{x}} \frac{s^2}{s^2+1}\,ds = \frac{(\sqrt{x})^2}{(\sqrt{x})^2+1} \cdot \frac{1}{2}x^{-\frac{1}{2}} = \frac{x}{x+1} \cdot \frac{1}{2\sqrt{x}}$

$= \boxed{\frac{x}{2\sqrt{x}(x+1)}}$

h) $\int_{-5}^{\cos x} t\cos(t^3)\,dt$

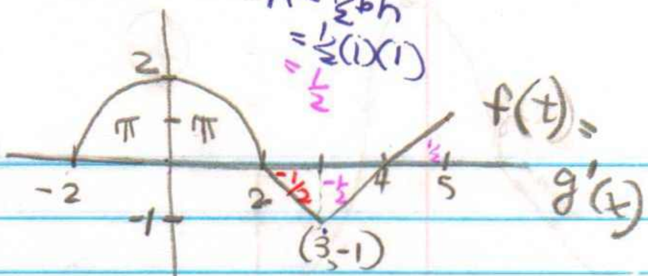
$= \boxed{\cos x \cdot \cos(\cos^3 x) \cdot -\sin x}$

i) $\int_{\tan x}^{17} \sin(t^4)\,dt = -\int_{17}^{\tan x} \sin(t^4)\,dt$

$= \boxed{-\sin(\tan^4 x) \cdot \sec^2 x}$

$A = \pi r^2$ $\quad \pi(2)^2 = 4\pi$

$\frac{1}{4}$ circle $= \frac{1}{4}\pi r^2$



② $g(x) = \int_0^x f(t)\,dt$

ⓐ find $g(0)$, $g(3)$, $g(-2)$ & $g(5)$

$g(0) = \int_0^0 f(t)\,dt = \boxed{0}$

$g(3) = \int_0^3 f(t)\,dt = \boxed{\pi - \frac{1}{2}}$

$g(-2) = \int_0^{-2} f(t)\,dt = -\int_{-2}^0 f(t)\,dt = \boxed{-\pi}$

$g(5) = \int_0^5 f(t)\,dt = \pi - 1 + \frac{1}{2} = \boxed{\pi - \frac{1}{2}}$

ⓑ at $\boxed{x = 2}$ ($g'(x) = 0$ crit #) & graph goes from pos to neg at $x = 2$

ⓒ Look at $x = -2, \overset{max}{2}, 4, 5$

abs max $g(2) = \int_0^2 f(t)\,dt = \pi$

$g(4) = \int_0^4 f(t)\,dt = \pi - 1$

$g(5) = \pi - \frac{1}{2}$

abs. min $g(-2) = -\pi$ ⟵ Abs min

(abs. min at $x = -2$)

ⓓ $g(\overset{x}{3}) = \overset{y}{\pi - \frac{1}{2}}$ $\quad g'(3) = \overset{slope}{-1}$ $\quad$ So $\boxed{y - \pi + \frac{1}{2} = -1(x - 3)}$

ⓔ pts of inflection at $x = 0$ $\quad x = 3$ $\quad$ Since $f'(x)$ goes from inc to dec. & vice versa

~~$x = 2$~~ → No sign change

ⓕ $[-\pi, \pi]$

③ $g(x) = \int_0^x f(t)\,dt$

ⓐ $g(0) = \int_0^0 f(t)\,dt = \boxed{0}$

$g(1) = \int_0^1 f(t)\,dt = \boxed{2}$

$g(2) = \int_0^2 f(t)\,dt = 2 + 2 + 1 = \boxed{5}$

$g(6) = \int_0^6 f(t)\,dt = 2 + 3 + 2 - 4 = \boxed{3}$

$g(3) = \boxed{7}$ $\qquad g(7) = 7 - 5 = \boxed{2}$

ⓑ g increasing where $g'(x)$ is pos. so

inc. $(0,3)$ $\quad$ dec. $(3,7)$

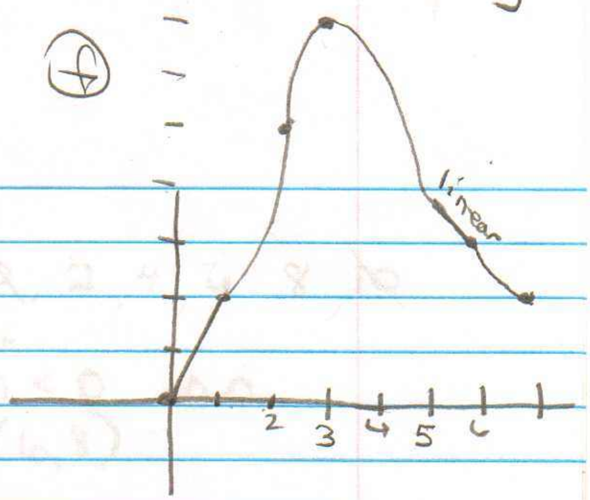
$f'(x) > 0$ $\quad f'(x) < 0$

ⓒ max value at $f'(x) = 0$ and goes from pos to neg.

so at $x = 3$

$g(3) = 7$ → so max $\boxed{(3,7)}$

(f)



(3d) min value at $x=0$

$\boxed{g(0)=0}$ min

(e) use pts found

$g(0)=0$ $g(6)=-3$

$g(1)=2$

$g(2)=5$

(4) $g(x)=\int_{-3}^{x} f(t)\,dt$ odd fcn →

(a) $g(-3) = \int_{-3}^{-3} f(t)\,dt = \boxed{0}$ (b) $g(3)=\int_{-3}^{3} f(t)\,dt = \boxed{0}$

(b) inc $(-3,0)$ b/c $f'(x)>0$

(c) max at $x=0$ b/c $f'(x)=0$ & goes from + to −

(d) min at $x=-3$ & 3 No rel min

(e) inflection pts at $x=-1$ & $x=1$ (rel min $f'(x)$ & rel max

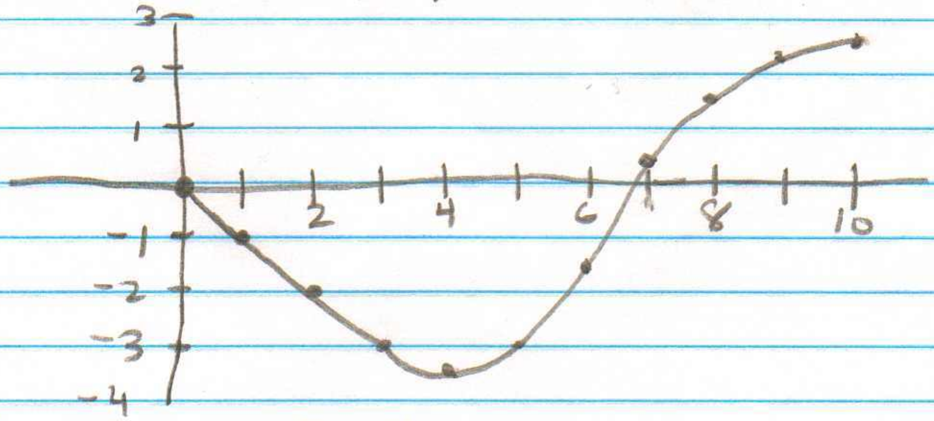
(5) $g(x)=\int_0^x f(t)\,dt$

use graph

$\int_0^0$

x	0	1	2	3	4	5	6	7	8	9	10
g(x)	0	−1	−2	−3	$-3\frac{1}{2}$	−3	−1.5	.25	1.5	2.25	2.5

(b)



(c) min at $x=4$

(d) four consec collinear $(0,0)$ $(1,-1)$ $(2,-2)$ $(3,-3)$

→ (e) inc. at greatest rate between 6, 7.

where $f''(t)$ is greatest ?

(b) $F(x) = \int_0^x f(t)\,dt$

(a) crit numbers are $x = \cancel{0}, 2, 4, 6, 8, \cancel{10}$

(b) decreasing on where $f(t)$ deriv. < 0 so
$(2, 4)$ and $(6, 8)$

(c) concave up where 2^{nd} deriv is positive
so from $(0, 1)$, $(3, 5)$ and $(7, 9)$

pts of inflection at $x = 1$, $x = 3$
$x = 5$, $x = 7$, and $x = 9$.