Diff EQ

1) $\frac{dy}{dx} = \frac{x^2}{y}$ given $(3,-5)$

$\int y\,dy = \int x^2\,dx$

$\frac{1}{2}y^2 = \frac{1}{3}x^3 + C$

$y^2 = \frac{2}{3}x^3 + C$

$y = \pm\sqrt{\frac{2}{3}x^3 + C}$

↑ choose neg because of given $(3,-5)$ ↓

$-5 = -\sqrt{\frac{2}{3}(3)^3 + C}$

$-5 = -\sqrt{18 + C}$

$25 = 18 + C$

$C = 7$

$$\boxed{y = -\sqrt{\frac{2}{3}x^3 + 7}}$$

2) $\frac{dy}{dx} = 6x^2y \quad y(0)=4$

$\int \frac{1}{y}\,dy = \int 6x^2\,dx$

$\ln|y| = 2x^3 + C$

$y = e^{2x^3+C} = Ce^{2x^3}$

(note: $e^C = C$)

$4 = Ce^{2(0)^3}$

$C = 4$

$$\boxed{y = 4e^{2x^3}}$$

3) $\frac{dy}{dx} = \frac{1}{y^2} \quad y(0)=4$

$\int y^2\,dy = \int 1\,dx$

$\frac{1}{3}y^3 = x + C$

$y^3 = 3x + C$

$y = \sqrt[3]{3x+C}$

$4 = \sqrt[3]{3(0)+C}$

$64 = 0 + C$

$C = 64$

$$\boxed{y = \sqrt[3]{3x+64}}$$

4) $\frac{dy}{dx} = \frac{1+x}{\sqrt{y}} \quad y(2)=9$

$\int \sqrt{y}\,dy = \int (1+x)\,dx$

$\frac{2}{3}y^{3/2} = x + \frac{x^2}{2} + C$

$y^{3/2} = \frac{3}{2}x + \frac{3}{4}x^2 + C$

$y = \left(\frac{3}{2}x + \frac{3}{4}x^2 + C\right)^{2/3}$

$9 = \left(\frac{3}{2}(2) + \frac{3}{4}(2)^2 + C\right)^{2/3}$

$C = 21$

$$\boxed{y = \left(\frac{3}{2}x + \frac{3}{4}x^2 + 21\right)^{2/3}}$$

5) $\frac{dy}{dx} = -xy^2 \quad y(1) = -0.25$

$\int \frac{1}{y^2}\,dy = -\int x\,dx$

$\frac{-1}{y} = \frac{-x^2}{2} + C$

$\frac{1}{y} = \frac{x^2 + C}{2}$

$y = \frac{2}{x^2+C}$

$-0.25 = \frac{2}{(1)^2+C}$

$\frac{-1}{4} = \frac{2}{1+C}$

$1 + C = -8$

$C = -9$

$$\boxed{y = \frac{2}{x^2-9}}$$

6) $\frac{dy}{dx} = \frac{4\sqrt{y}\ln x}{x} \quad y(e) = 9$

$\int \frac{1}{\sqrt{y}}\,dy = 4\int \frac{\ln x}{x}\,dx$ let $u = \ln x$, $du = \frac{1}{x}dx$, $\int u\,du$, $\frac{u^2}{2}$

$2y^{1/2} = 4\left(\frac{(\ln x)^2}{2}\right) + C$

$y^{1/2} = (\ln(x))^2 + C$

$y = ((\ln x)^2 + C)^2$

$9 = ((\ln e)^2 + C)^2$

$3 = (1)^2 + C$

$C = 2$

$$\boxed{y = ((\ln x)^2 + 2)^2}$$

7) $\frac{dy}{dx} = 4x^3y \quad (0,7)$

$\int \frac{1}{y}dy = \int 4x^3 dx$

$\ln|y| = x^4 + C$

$y = Ce^{x^4}$

$7 = Ce^{(0)^4}$

$C = 7$

$\boxed{y = 7e^{x^4}}$

8) $\frac{dy}{dx} = \frac{y^2}{x^3} \quad (1,1)$

$\int \frac{1}{y^2}dy = \int \frac{1}{x^3}dx$

$\frac{-1}{y} = \frac{-1}{2x^2} + C$

$\frac{1}{y} = \frac{1}{2x^2} + \frac{2x^2C}{2x^2} = \frac{1+Cx^2}{2x^2}$

$y = \frac{2x^2}{1+Cx^2}$

$1 = \frac{2(1)^2}{1+C(1)^2}$

$1 = \frac{2}{1+C}$

$1+C = 2$

$C = 1$

$\boxed{y = \frac{2x^2}{1+x^2}}$

9) $\frac{dy}{dt} = -3y \quad (0,1)$

$\int \frac{1}{y}dy = -3\int dt$

$\ln y = -3t + C$

$y = Ce^{-3t}$

$1 = Ce^{-3(0)}$

$C = 1$

$\boxed{y = e^{-3t}}$

$\frac{1}{3} = e^{-3t}$

$-3t = \ln\frac{1}{3}$

$t = \frac{-1}{3}\ln\frac{1}{3}$

$\boxed{t = 0.3662}$

10) $\frac{dy}{dx} = y\cos x \quad (0,3)$

$\int \frac{1}{y}dy = \int \cos x\, dx$

$\ln y = \sin x + C$

$y = Ce^{\sin x}$

$3 = Ce^{\sin(0)}$

$C = 3$

$\boxed{y = 3e^{\sin x}}$

11) $\frac{dy}{dx} = 2y$ where $y = f(x)$, $f(2) = 1$

$\int \frac{1}{y}dy = 2\int dx$

$\ln y = 2x + C$

$y = Ce^{2x}$

$f(x) = Ce^{2x}$

$1 = Ce^4$

$C = \frac{1}{e^4}$

$f(x) = \frac{1}{e^4}e^{2x}$

$\boxed{f(x) = e^{2x-4}}$

12) $\frac{dy}{dx} = 2y^2 \quad (1,-1)$

$\int \frac{1}{y^2}dy = 2\int dx$

$\frac{-1}{y} = 2x + C$

$y = \frac{-1}{2x+C}$

$-1 = \frac{-1}{2(1)+C}$

$2+C = 1$

$C = -1$

$\boxed{y = \frac{-1}{2x-1}}$

$y = \frac{-1}{2(2)-1} = \boxed{\frac{-1}{3}}$

B

13) $\frac{dy}{dx} = x^2y$

$\int \frac{1}{y}dy = \int x^2 dx$

$\ln y = \frac{1}{3}x^3 + C$

$y = Ce^{\frac{1}{3}x^3}$

C

14) $\frac{dN}{ds} = k(250-N)$

$\int \frac{1}{250-N}dN = \int k\, ds$

$-\ln|250-N| = ks + C$

$\ln(250-N) = -ks + C$

$250 - N = Ce^{-ks}$

$\boxed{N = 250 - Ce^{-ks}}$

15) $\frac{dR}{dt} = \frac{k}{\sqrt{R}}$

$\int \sqrt{R}\, dR = \int k\, dt$

$\frac{2}{3}R^{3/2} = kt + C$

$\boxed{R = \left(\frac{3}{2}kt + C\right)^{2/3}}$

16) $\frac{dy}{dx} = k\cdot x\cdot(L-y)$

$\int \frac{1}{L-y}dy = \int kx\, dx$

$-\ln(L-y) = \frac{kx^2}{2} + C$

$L - y = Ce^{\frac{-kx^2}{2}}$

$\ln(L-y) = \frac{-kx^2}{2} + C$

$\boxed{y = L - Ce^{\frac{-kx^2}{2}}}$