

## Average Value

$$\textcircled{1} f(x) = (x-3)^2 \cdot [2, 5]$$

$$\textcircled{a} \frac{1}{5-2} \int_2^5 (x-3)^2 dx = \frac{1}{3} \int_2^5 (x^2 - 6x + 9) dx$$

$$= \frac{1}{3} \left[ \frac{x^3}{3} - 3x^2 + 9x \right]_2^5 = \frac{1}{3} \left[ \left( \frac{125}{3} - 75 + 45 \right) - \left( \frac{8}{3} - 12 + 18 \right) \right]$$

$$= \frac{1}{3} \left[ \frac{35}{3} - \frac{26}{3} \right] = \frac{1}{3} \left[ \frac{9}{3} \right] = \boxed{1}$$

$\textcircled{b}$  Find  $c$  such the  $f_{\text{ave}} = f(c)$  so  $f(c) = 1$

$$(x-3)^2 = 1$$

$$x-3 = \pm 1$$

$$x = 3 \pm 1 = \boxed{4 \text{ or } 2}$$

$$\textcircled{2a} f(x) = \sqrt{x}; [0, 4] \quad \sqrt{x} = x^{1/2}$$

$$\frac{1}{4-0} \int_0^4 \sqrt{x} dx = \frac{1}{4} \left[ \frac{2}{3} x^{3/2} \right]_0^4 = \frac{1}{4} \left[ \frac{2}{3} (8) - 0 \right] = \frac{1}{4} \cdot \frac{16}{3} = \boxed{\frac{4}{3}}$$

$$\textcircled{b} \frac{4}{3} = \sqrt{x}$$

$$\boxed{x = \frac{16}{9}}$$

$\textcircled{3}$ $x$	20	25	30	35	40	45	50
$f(x)$	42	38	31	29	35	48	60

3 divisions

midpt

$$\Delta x = 10$$

Riemann

$$10(38 + 29 + 48) = 10(115) = 1150$$

$$f_{\text{ave}} = \frac{1}{50-20} \int_{20}^{50} f(x) dx \quad \text{Avg value}$$

$$\frac{1}{30} (1150) = \boxed{38.33\bar{3}}$$

Estimate

- ④ avg velocity during 1<sup>st</sup> 12 seconds by midpt Riemann  $\Delta x = 4$

$$\frac{1}{12-0} [4(20+50+66)] \approx \frac{1}{2} [544] = 45\frac{1}{2}$$

$$\approx \boxed{45.333 \text{ km/hr}}$$

used graph →

- ④ when instantaneous velocity equal to avg velocity? at about 5 seconds.

$$\textcircled{5} \frac{1}{12-0} \int_0^{12} (50 + 14 \sin(\frac{\pi t}{12})) dt = \frac{1}{12} (706.952)$$

$$= \boxed{58.913^\circ \text{F}}$$

$$\textcircled{6} T(t) = 20 + 75e^{-t/50}$$

$$\frac{1}{30-0} \int_0^{30} (20 + 75e^{-t/50}) dt = \frac{1}{30} [2291.956]$$

$$= \boxed{76.399^\circ \text{C}}$$

- ⑦  $t=0 \rightarrow$  Jan 1, 2010 cost  $C(t)$  in dollars per day

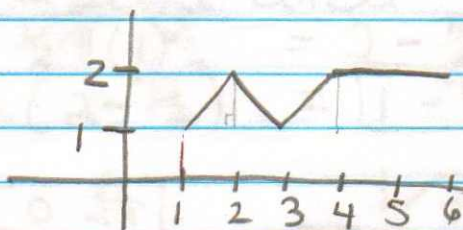
Interpret  $\int_0^{90} C(t) dt$  is Total Cost in dollar for 1<sup>st</sup> 90 days to heat house of 2010  $\rightarrow$  cost from Jan 1 to Mar 31

$\frac{1}{90-0} \int_0^{90} c(t) dt$  avg cost of heating house per day for 1<sup>st</sup> Quarter of 2010.

⑧

$$\textcircled{a} \int_1^6 f(x) dx$$

$$= \frac{1}{2}(1)(2) + \frac{1}{2}(1)(1) + 2(1) = 1 + \frac{1}{2} + 2 = \boxed{8\frac{1}{2} \text{ sq units}}$$



④ avg value  $\frac{1}{6-1} \int_1^6 f(x) dx = \frac{1}{5} (17\frac{1}{2}) = \boxed{\frac{17}{10} \approx 1.7}$

Given:  $\frac{1}{5} \int_1^6 f(x) dx = 4$        $\frac{1}{2} \int_6^8 f(x) dx = 5$

9) avg value of  $y = f(x) = 4$  for  $1 \leq x \leq 6$   
 $= 5$  for  $6 \leq x \leq 8$

What is avg value of  $f(x)$  for  $1 \leq x \leq 8$

$$\int_1^6 f(x) dx + \int_6^8 f(x) dx = \int_1^8 f(x) dx$$

$$\frac{1}{5} (20) + \frac{1}{2} (10) = \frac{1}{7} (30)$$

$$4 + 5 = \boxed{\frac{30}{7} \text{ or } 4.286}$$

10) Suppose  $\int_0^3 f(x) dx = 6$

a) avg value of  $f(x)$  on  $[0, 3]$        $\frac{1}{3-0} \int_0^3 f(x) dx = \frac{1}{3}(6) = \boxed{2}$

b)  $f(x)$  is even

What is  $\int_{-3}^3 f(x) dx = \boxed{12}$

avg value =  $\frac{1}{3-(-3)}(12) = \boxed{2}$

c)  $f(x)$  is odd

What is  $\int_{-3}^3 f(x) dx = \boxed{0}$

avg value =  $\frac{1}{6}(0) = \boxed{0}$

11)  $f(x) = \begin{cases} x+4 & -4 \leq x \leq -1 \\ -x+2 & -1 \leq x \leq 2 \end{cases}$  on  $[-4, 2]$

avg value  $\frac{1}{2-(-4)} \int_{-4}^2 f(x) dx = \frac{1}{6} (\frac{1}{2}(3)(6)) = \frac{1}{6}(9) = \boxed{\frac{3}{2}}$

12)  $f(x) = 1 - \sqrt{1-x^2}$  on  $[-1, 1]$

Area = rectangle - semi circle

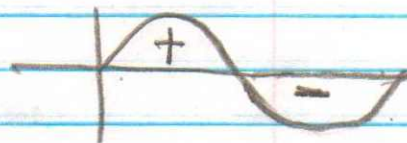
$$= 2(1) - \frac{1}{2}\pi(1)^2 = 2 - \frac{\pi}{2}$$

$$\frac{1}{1-(-1)} \int_{-1}^1 f(x) dx = \frac{1}{2} (2 - \frac{\pi}{2}) = \boxed{1 - \frac{\pi}{4}}$$

13)  $f(x) = \sin x$ ,  $[0, 2\pi]$

$$\frac{1}{2\pi} \int_0^{2\pi} f(x) dx = \frac{1}{2\pi}(0)$$

$$= \boxed{0}$$

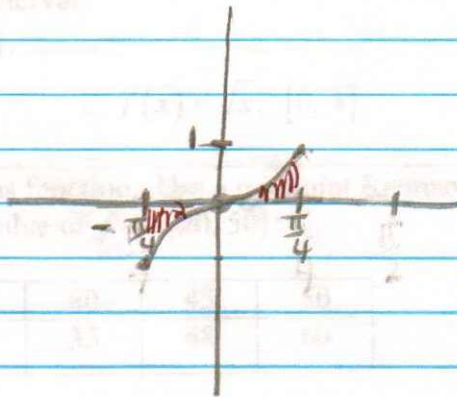


ON AVERAGE VALUE

(14)  $f(x) = \tan x \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

odd fun

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan x \, dx = 0$$



$$\frac{1}{\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)} (0) = \boxed{0}$$