CALCULUS
WORKSHEET ON DERIVATIVES 2nd derivative test

Work the following on notebook paper except for problems 11 – 12. Do not use your calculator. On problems 1 – 4, find the critical points of each function, and determine whether they are relative maximums or relative minimums by using the Second Derivative Test whenever possible.

1. $f(x) = 3x^3 - 36x - 3$

2. $f(x) = x(x-2)^2 + 1$

3. $f(x) = \sin x - \cos x,\ 0 \le x \le 2\pi$

4. $f(x) = \left(x^2 - 4\right)^3$

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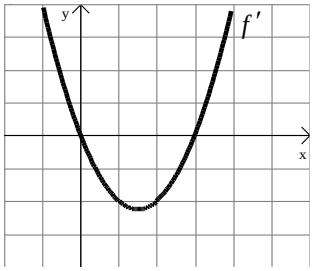
5. Suppose that the function f has a continuous second derivative for all x and that $f(0) = 2,\ f'(0) = -3,\ f''(0) = 4$. Let g be a function whose derivative is given by $g'(x) = \left(x^3 + 1\right)\left(3f(x) + 2f'(x)\right)$ for all x. Does g have a local maximum or a local minimum at $x = 0$? Justify your answer.

On problems 6 – 7, the graph of the derivative, f', of a function f is shown.
(a) On what interval(s) is f increasing or decreasing? Justify your answer.
(b) At what value(s) of x does f have a local maximum or local minimum? Justify your answer.

6.



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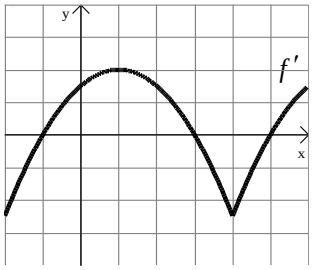

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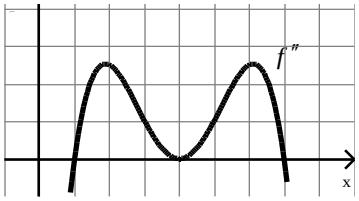
8. The graph of the second derivative, f'', of a function f is shown. State the x-coordinates of the inflection points of f. Justify your answer.



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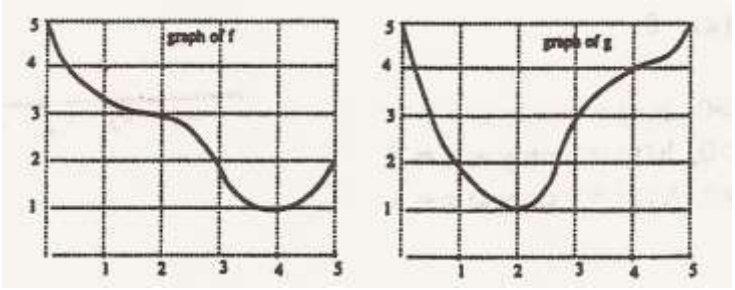
9. For what values of a and b does the function $f(x) = x^3 + ax^2 + bx + 2$ have a local maximum when $x = -3$ and a local minimum when $x = -1$?

10. The function h is defined by $h(x) = f(g(x))$, where f and g are the functions whose graphs are shown below.



(a) Evaluate $h(2)$.

(b) Estimate $h'(1)$.

(c) Is the graph of the composite function h increasing or decreasing at $x = 3$? Show your reasoning.

(d) Find all values of x for which the graph of h has a horizontal tangent. Show your reasoning.

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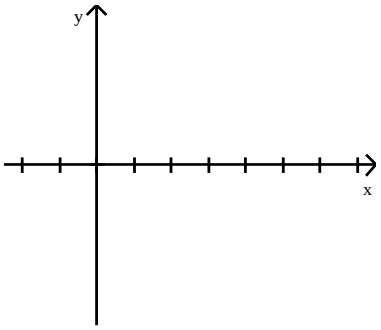
11. Sketch a graph of a differentiable function $f(x)$ over the closed interval $[-2, 7]$, where $f(-2) = f(7) = -3$ and $f(4) = 3$. The roots of $f(x)$ occurs at $x = 0$ and $x = 6$, and $f(x)$ has the properties indicated in the table below.

x	$-2 < x < 0$	$x = 0$	$0 < x < 2$	$x = 2$	$2 < x < 4$	$x = 4$	$4 < x < 7$
$f'(x)$	positive	0	positive	1	positive	0	negative
$f''(x)$	negative	0	positive	0	negative	0	negative



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12. Sketch the function $h(x)$ from the following information:

(a) The domain of h is $(-\infty, \infty)$.

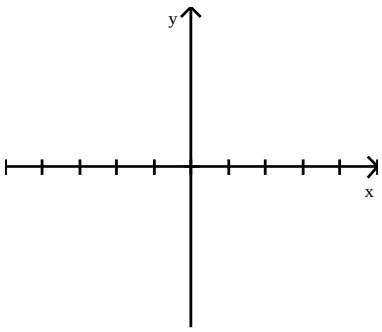
(b) $\lim_{x \to 0^+} h(x) = \infty$

(c) $\lim_{x \to \infty} h(x) = 0$

(d) For $x > 0$, $h(x) = 0$ only at $x = 1$.

(e) For $x > 0$, $h'(x) = 0$ only at $x = 2$.

(f) For $x > 0$, $h''(x) = 0$ only at $x = 3$.



Answers to Worksheet on Second Derivative Test

1. Rel. max. at $(-2, 45)$, rel. min. at (2, - 51)

2. Rel. max. at $\left(\frac{2}{3}, \frac{59}{27}\right)$, rel. min. at (2, 1)

3. Rel. max. at $\left(\frac{3\pi}{4}, \sqrt{2}\right)$, rel. min. at $\left(\frac{7\pi}{4}, -\sqrt{2}\right)$

4. Rel. min. at $(0, -64)$; neither at $(-2, 0)$ or at (2, 0)

5. local max.

6. (a) incr. on $(-\infty, 0) \cup (3, \infty)$; decr. on (0, 3) (b) Rel. max. at $x = 0$, rel. min. at $x = 3$

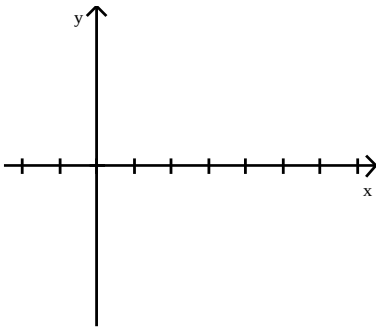
7. (a) decr. on $(-\infty, -1) \cup (3, 5)$; incr. on $(-1, 3) \cup (5, \infty)$
   (b) Rel. min. at $x = -1$, $x = 5$; rel. max. at $x = 3$

8. $x = 1$ and $x = 7$

9. $a = 6$, $b = 9$

10. (a) 3.4 (b) $\frac{1}{4}$ (c) decr. (d) 2, 0.25, 4

11.



12.