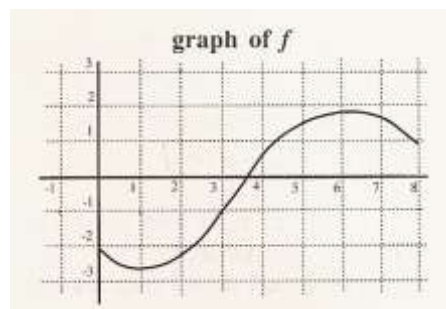
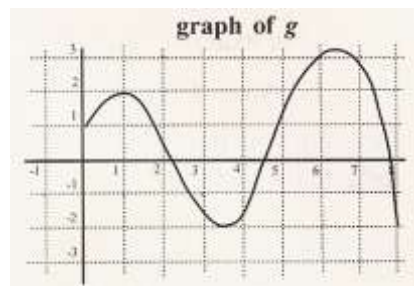


CALCULUS  
WORKSHEET ON THE MEAN VALUE THEOREM

1. Use the graph of  $f$  to estimate the numbers in  $[0, 8]$  that satisfy the conclusion of the Mean Value Theorem.



2. Use the graph of  $g$  to estimate the numbers in  $[0, 8]$  that satisfy the conclusion of the Mean Value Theorem.



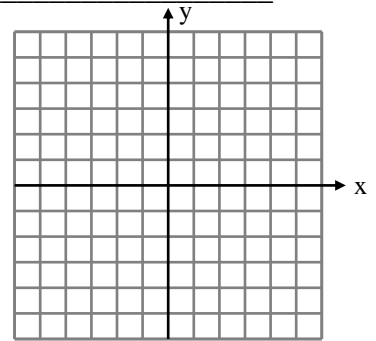
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In problems 3 – 10, determine whether  $f$  satisfies the hypotheses of the Mean Value Theorem on the interval  $[a, b]$ . If it does, find all numbers  $c$  in  $(a, b)$  such that  $f'(c) = \frac{f(b) - f(a)}{b - a}$ .

3.  $f(x) = 3x^2 + x - 4$   $[1, 5]$
4.  $f(x) = x^{2/3}$   $[-1, 2]$
5.  $f(x) = \cos x - \sin x$   $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$
6.  $f(x) = 1 - |x|$   $[-1, 2]$
7.  $f(x) = \frac{x+1}{x-1}$   $[-3, 0]$
8.  $f(x) = x^{1/3}$   $[0, 27]$
9.  $f(x) = \sqrt{x}$   $[4, 9]$
10.  $f(x) = 2\sin x + \sin(2x)$   $[0, \pi]$

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11. Sketch a graph of the function  $f$  if  $f(x) = \begin{cases} x+2, & x \leq 1 \\ x^2, & x > 1 \end{cases}$ .

Show that  $f$  fails to satisfy the hypothesis of the Mean Value Theorem on the interval  $[-2, 2]$ , but the conclusion of the theorem is still valid.



12. A trucker handed in a ticket at a toll booth showing that in 2 hours he had covered 159 miles on a toll road with speed limit 65 mph. The trucker was cited for speeding. Why?
13. It took 20 sec for the temperature to rise from  $0^\circ\text{F}$  to  $212^\circ\text{F}$  when a thermometer was taken from a freezer and placed in boiling water. Explain why at some moment in that interval the mercury was rising at exactly  $10.6^\circ\text{F}/\text{sec}$ .
14. A marathoner ran the 26.2-mi New York City Marathon in 2.2 h. Show that at least twice, the marathoner was running at exactly 11 mph.