If Continuous @ $x=c$

(i) $f(c)$ defined

(ii) $\lim_{x \to c} f(x)$ exist

(iii) $\lim_{x \to c} f(x) = f(c)$

Chapter 1 Problems

Find the limit of the following expressions

1) $\lim_{x \to 2} x+3 = 5$

2) $\lim_{x \to 13} \sqrt{x+3} = 4$

3) $\lim_{x \to 2} \frac{x-2}{x^2-4}$

$\lim_{x \to 2} \left[\frac{(x-2)}{(x+2)(x-2)}\right] = \frac{1}{4}$

4) $\lim_{x \to 2} \left[\frac{(x-2)}{2-\sqrt{x+2}} \cdot \frac{(2+\sqrt{x+2})}{2+\sqrt{x+2}}\right] = \frac{(x-2)(2+\sqrt{x+2})}{4-(x+2)} = -\left[\frac{(x-2)(2+\sqrt{x+2})}{(2-x)}\right] = -4$

5) $\lim_{x \to 0} \left[\frac{\sin x}{3x}\right] = \frac{1}{3} \frac{\sin x}{x}$

$\left(\frac{1}{3}\right)(1) = \frac{1}{3}$

6) $\lim_{x \to 0} \frac{1-\cos x}{x} = 0$

7) $\lim_{x \to 5} \frac{1}{x-5} = DNE$

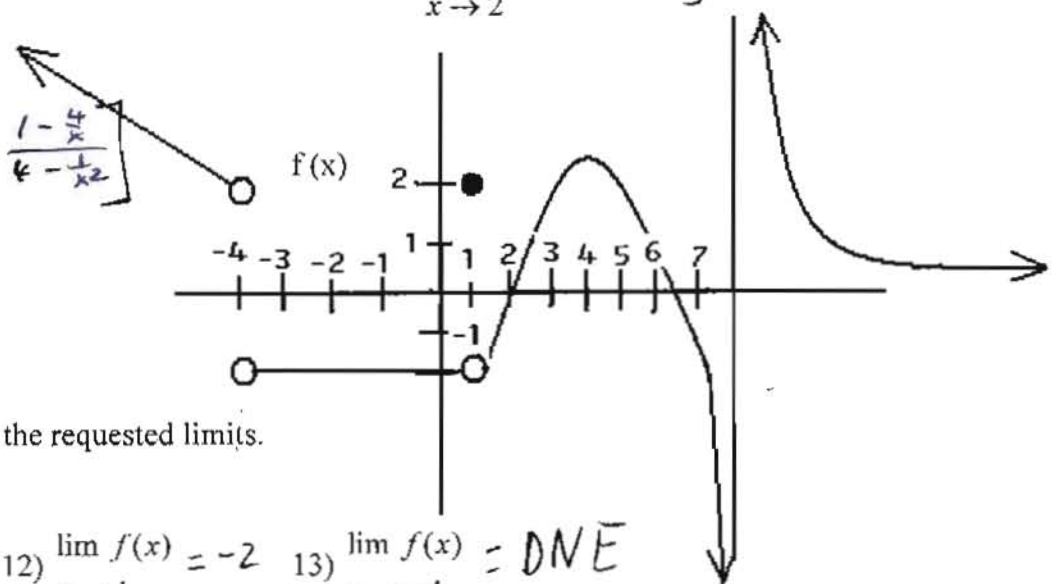
$\lim_{x \to 5^+} \frac{1}{x-5} = \frac{1}{+0} = \infty \quad \lim_{x \to 5^-} \frac{1}{x-5} = \frac{1}{-0} = -\infty$

8) $\lim_{x \to \infty} \frac{4}{x+3} = 0$

9) $\lim_{x \to -\infty} x+3 = -\infty$

10) $\lim_{x \to 2} f(x) = 4 \quad \lim_{x \to 2} g(x) = 3 \quad h(x) = \frac{2f(x)}{g(x)}$ *find* $\lim_{x \to 2} h(x) = \frac{8}{3}$

10) $\lim_{x \to -\infty} \left[\frac{(x^2-4x)x^2}{(4x^2-1)x^2}\right] = \frac{1-\frac{4}{x}}{4-\frac{1}{x^2}}$



Given the sketch, find the requested limits.

11) $\lim_{x \to \infty} f(x) = 0$

12) $\lim_{x \to 1} f(x) = -2$

13) $\lim_{x \to -4} f(x) = DNE$

14) $\lim_{x \to 8} f(x) = DNE$

15) $\lim_{x \to -4^-} f(x) = 2$

16) $\lim_{x \to -4^+} f(x) = -2$

17) $f(-4) =$ undefined

18) $f(1) = 2$

19) $f(8) =$ und

20) $\lim_{x \to 2} 2f(x)-1 = -1$

$2(0)-1$

21) $\lim_{x \to 8^+} f(x) = \infty$

22) $f(x)=\begin{cases}2x^2-1 & x>1\\ ax-3 & x\le 1\end{cases}$ Solve for the correct values of a and b so that f(x) is continuous.

$2(1)^2-1=a(1)-3$ $\qquad 1=a-3$

$a=4$

23) P(t) is the function that models the population of Sugar Land. P(t) is a continuous function. The population in 1990 was 60 thousand. The population in 2010 was 120 thousand. How can we justify that at some point between 1990 and 2010 the population crossed 100 thousand?

P(t) is continuous. $P(1990)=60$ $P(2010)=120$

100 is on $[60,120]$

∴ IVT

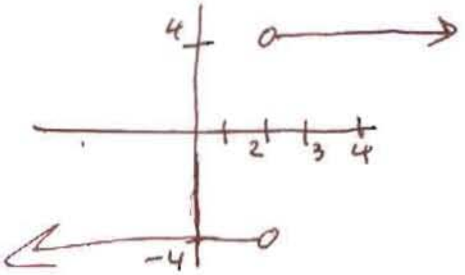
24) Prove that f(x) is continous at x = 5. $f(x)=\begin{cases}\dfrac{x-5}{x^2-4x-5} & x\neq 5\\ \dfrac{1}{\sqrt{31+x}} & x=5\end{cases}$

$\lim\limits_{x\to 5}\left[\dfrac{(x-5)}{(x-5)(x+1)}=\dfrac{1}{x+1}\right]=\dfrac{1}{6}$

① $f(5)$ is defined $\left(\frac{1}{6}\right)$

② $\lim\limits_{x\to 5} f(x)$ exists $\left(\frac{1}{6}\right)$ ③ $\lim\limits_{x\to 5} f(x)=\frac{1}{6}=f(5)$

25) $\lim\limits_{x\to 2}\dfrac{4|x-2|}{x-2}=$ DNE



26) $\lim\limits_{x\to\infty}\dfrac{x^2-4}{3^x}=\boxed{0}$

Denominator is increasing more rapidly.

27) $f(x)=\begin{cases}3x-5 & x\neq 1\\ 2 & x=1\end{cases}$

Is this function continuous at x=1

No