Chapter 2 Review Packet Part 1

Definition of the Derivative of a Function

The **derivative** of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

provided the limit exists. For all x for which this limit exists, f' is a function of x.

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Alternative Version of a derivative

THEOREM 2.1 Differentiability Implies Continuity

If f is differentiable at $x = c$, then f is continuous at $x = c$.

Summary of Differentiation Rules

General Differentiation Rules	Let f, g, and u be differentiable functions of x.		
	Constant Multiple Rule: $\frac{d}{dx}[cf] = cf'$	*Sum or Difference Rule*: $\frac{d}{dx}[f \pm g] = f' \pm g'$	
	Product Rule: $\frac{d}{dx}[fg] = fg' + gf'$	*Quotient Rule*: $\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{gf' - fg'}{g^2}$	
Derivatives of Algebraic Functions	*Constant Rule*: $\frac{d}{dx}[c] = 0$	*(Simple) Power Rule*: $\frac{d}{dx}[x^n] = nx^{n-1}$, $\frac{d}{dx}[x] = 1$	
Derivatives of Trigonometric Functions	$\frac{d}{dx}[\sin x] = \cos x$	$\frac{d}{dx}[\tan x] = \sec^2 x$	$\frac{d}{dx}[\sec x] = \sec x \tan x$
	$\frac{d}{dx}[\cos x] = -\sin x$	$\frac{d}{dx}[\cot x] = -\csc^2 x$	$\frac{d}{dx}[\csc x] = -\csc x \cot x$
Chain Rule	*Chain Rule*: $\frac{d}{dx}[f(u)] = f'(u)\,u'$	*General Power Rule*: $\frac{d}{dx}[u^n] = nu^{n-1}\,u'$	

1) Let f be the function defined by $f(x)=\begin{cases}\sqrt{x+1} & 0\le x\le 3\\ 5-x & 3<x\le 5\end{cases}$

a) Is f continuous at x=3. Explain why or why not.
b) Find the slope of the line tangent to f(x) when x = 2

c) Suppose the function g is defined by $g(x)=\begin{cases}k\sqrt{x+1} & 0\le x\le 3\\ mx+2 & 3<x\le 5\end{cases}$ where k and m are constants. If g is differentiable at x = 3, what are the values of k and m?

2) What is $\lim\limits_{h\to 0}\dfrac{\cos\left(\dfrac{3\pi}{2}+h\right)-\cos\left(\dfrac{3\pi}{2}\right)}{h}$

3) If f(x) = $\sin^2(3 - x)$ then f ‘ (0) =
a) -2 cos 3 b) -2 sin 3 cos 3 c) 6 cos 3 d) 2 sin 3 cos 3 e) 6 sin 3 cos 3

4) A differentiable function f has the property that f(5) = 3 and f ‘ (5) = 4. What is the estimate for f(4.8) using the local linear approximation for f at x = 5?

5) What is $\lim\limits_{h\to 0}\dfrac{e^{3+h}-e^{3}}{h}$

6) Sketch and write the equations of three common types of functions that aren’t differentiable at at least one point but are continuous at all points.

7) $f(x)=x^3+7x$ Find the equation of the line tangent to the function at x = 1

8) Water is flowing into a tank over a 24-hour period. The amount of water $W(t)$ in the tank at various times is measured, and the results are given in the table below, where $W(t)$ is measured in gallons and t is measured in hours. There are 150 gallons of water in the tank at t = 0.

t (hours)	0	4	8	12	16	20	24
$W(t)$ (gallons)	150	184	221	257	294	327	357

(a) Use data from the table to find $W(8)$. Using appropriate units, explain the meaning of your answer.
(b) For $0<t<24$, must there be a time t when the tank contains 265 gallons of water? Justify your answer.
(c) Use data from the table to find an approximation for $W'(15)$. Show the computations that lead to your answer. Using appropriate units, explain the meaning of your answer

9) $\frac{d}{dx}(x\sin x+\cos x)=$

10) Find the equations of the tangent lines to the graph of $f(x)=\frac{x+1}{x-1}$ that are parallel to the line 2y + x = 6 then graph the function and its tangent lines.