Calculus AB Chapter 2 Review

A

① $f(x) = \begin{cases} \sqrt{x+1} & ; 0 \le x \le 3 \\ 5-x & ; 3 < x \le 5 \end{cases}$

ⓐ $\sqrt{3+1} = 5-3$

$2 = 2$

ⓑ

$$f'(x) = \begin{cases} \frac{1}{2}(x+1)^{-\frac{1}{2}} & ; 0 \le x \le 3 \\ -1 & ; 3 < x \le 5 \end{cases}$$

$$f'(2) = \frac{1}{2\sqrt{3}}$$

ⓒ

continuous:

$k\sqrt{3+1} = 3m+2$

$2k = 3m+2$

diff:

$m = \frac{k}{2}(3+1)^{-\frac{1}{2}}$

$m = \frac{k}{4}$

$$2k = \frac{3k}{4} + 2$$

$$8k = 3k + 8$$

$$5k = 8$$

$$k = \frac{8}{5} \qquad m = \frac{2}{5}$$

② $$\lim_{h \to 0} \frac{\cos\left(\frac{3\pi}{2}+h\right) - \cos\frac{3\pi}{2}}{h} = \frac{d}{dx}\cos x \Big|_{x=\frac{3\pi}{2}}$$

$$= -\sin\frac{3\pi}{2} = \boxed{1}$$

③ $f(x) = \sin^2(3-x)$

$f'(x) = 2\sin(3-x)\cos(3-x)(-1)$

$f'(0) = -2\sin 3\cos 3$ ⓑ

(4) $f(5)=3 \quad f'(5)=4$

equation of tangent line:

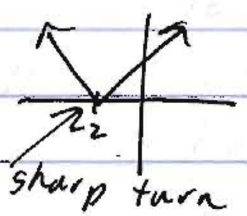
$$y-3=4(x-5)$$
$$y=4(x-5)+3$$
$$=4(4.8-5)+3$$
$$=4(-0.2)+3=-0.8+3$$
$$f(4.8)\approx 2.2$$

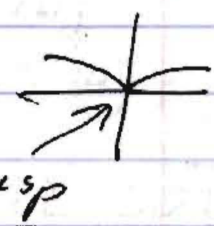
(5) $$\lim_{h\to 0}\frac{e^{(3+h)}-e^{3}}{h} = \frac{d}{dx}e^{x}\Big|_{x=3}$$

$$\boxed{e^{3}}$$

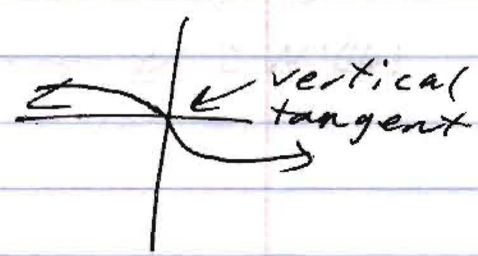
(6) $y=|x+2|$ $\qquad$ $y=x^{2/3}$

A.M.V.





$y=\sqrt[3]{x}$



(7) $f(x)=x^{3}+7x \qquad f(1)=8$

$f'(x)=3x^{2}+7 \qquad f'(1)=10$

$$y-8=10(x-1)$$

8(a) $w'(8) \approx \frac{257 - 184}{12 - 4} \approx 9.125$

use Ave. Rt. of change

(b) $W(0) = 150$

$W(24) = 357$

- function is continuous.
- $150 < 265 < 357$
- there must be a t value on $[0, 24]$ where $W(t) = 265$ by I.V.T.

(c)

Ave. Rt of change on $[12, 16]$

$\frac{294 - 257}{16 - 12} = 9.25 \leftarrow \approx w'(14)$

Ave Rt of change on $[12, 20]$

$\frac{327 - 257}{20 - 12} = 8.75 \leftarrow \approx w'(16)$

$\approx$ 9

(a) $\frac{d}{dx}(x\sin x + \cos x) = \cancel{\sin x} + x\cos x - \cancel{\sin x}$

$= x\cos x$

10 $f(x) = \frac{x+1}{x-1}$

$f'(x) = \frac{(x-1) - (x+1)}{(x-1)^2} = \frac{-2}{(x-1)^2}$

$\frac{-2}{(x-1)^2} = -\frac{1}{2}$

$(x-1)^2 = 4$

$x - 1 = \pm 2$

$x = 1 \pm 2 = 3$ or -1

$2y + x = 6$

$y = -\frac{1}{2}x + 3$

$f(3) = 2$

$f(-1) = 0$

$y - 2 = -\frac{1}{2}(x-3)$

$y = -\frac{1}{2}(x+1)$