

Notes for Implicit Differentiation

Guidelines for Implicit Differentiation

1. Differentiate both sides of the equation *with respect to x*.
2. Collect all terms involving dy/dx on the left side of the equation and move all other terms to the right side of the equation.
3. Factor dy/dx out of the left side of the equation.
4. Solve for dy/dx .

Solve for $\frac{dy}{dx}$

$$2xy + y^2 = x$$

$$\frac{d}{dx}[2xy + y^2 = x]$$

$$2x \frac{dy}{dx} + 2y + 2y \frac{dy}{dx} = 1 \quad \text{product rule}$$

$$\frac{dy}{dx} 2x + 2y = 1 - 2y$$

$$\frac{dy}{dx} = \frac{1 - 2y}{2x + 2y}$$

Derivative WS Part 2

1) Consider the curve $y^2 = 2 + xy$

a) Solve for $\frac{dy}{dx}$

b) Find all points (x,y) on the curve where the line tangent to the curve has slope of $\frac{1}{2}$

c) Show that there are no points (x,y) on the curve where the line tangent to the curve is horizontal.

2) Consider the curve given by the equation $xy^2 - x^3y = 6$

a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$

b) Find all points on the curve whose x-coordinate is 1 and write an equation for the tangent line at each of these points.

c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

3) Given the following information, answer questions a-d

$$f(2) = 3 ; f'(2) = 4 ; g(2) = 5 ; g'(2) = -1$$

a) $h(x) = \frac{g(x)}{f(x)}$ $h'(2) =$ _____ b) $h(x) = g(x) \cdot f(x)$ $h'(2) =$ _____

c) $h(x) = g(x) - f(x)$ $h'(2) =$ _____ d) $f^{-1}(3) =$ _____

4) $f(x) = x\sqrt{2x} + \cos x$ a) Find $f'(x) =$

b) Find the equation of the line tangent to the function $f(x)$ when $x = \pi$

5) Assuming that $f(x)$ is differentiable at $x = 5$, which of the following statements are always true.

I) $f(5)$ is a real number.

II) There is a sharp turn at $x = 4$

III) If $f(5) = 2$ then the limit of $f(x)$ as x approaches 5 must be 2.

IV) $f(x)$ is continuous at $x = 5$

6) Find the slope of the tangent line when $x = 2$ on the equation of the ellipse that is 10 units wide and 4 units high.

7) $\frac{d}{dx} \cos^2(x^3) =$

a) $6x^2 \sin(x^3) \cos(x^3)$

b) $6x^2 \cos(x^3)$

c) $\sin(x^3)$

d) $-6x^2 \sin(x^3) \cos(x^3)$

e) $-2 \sin(x^3) \cos(x^3)$

8) At what point on the graph of $y = 0.5x^2$ is the tangent line parallel to $2x - 4y = 3$

a) $(0.5, -0.5)$

b) $(0.5, 0.125)$

c) $(1, -0.25)$

d) $(1, 0.5)$

e) $(2, 2)$