① $y^2 = 2 + xy$ Find $\frac{dy}{dx}$

ⓐ $2y\frac{dy}{dx} = x\frac{dy}{dx} + y$

$$\frac{dy}{dx} = \frac{y}{2y - x}$$

ⓑ $\frac{1}{2} = \frac{y}{2y-x}$ $\quad 2y - x = 2y$

$-x = 0$

$x = 0$

$y^2 = 2 + 0y$

$y^2 = 2$

$y = \pm\sqrt{2}$ $\quad (0, \sqrt{2})$

$(0, -\sqrt{2})$

ⓒ $0 = \frac{y}{2y-x}$ $\rightarrow$ $0^2 = 2 + 0x$

$y = 0$ $\quad 0 = 2$ can't happen

no pts w/ $y=0$

② $xy^2 - x^3y = 6$

$2xy\frac{dy}{dx} + y^2 - x^3\frac{dy}{dx} - 3x^2y = 0$

ⓐ $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$

ⓑ $1y^2 - 1y = 6$ $\quad 9x - 3x^3 = 6$

$(y-3)(y+2) = 0$ $\quad x^3 - 3x + 2 = 0$

$y = 3$ or $y = -2$

$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$(1, 3)$ $(-2, 3)$

$4x + 2x^3 = 6$

$x^2 + x - 2 = 0$ $\quad x = 1$ OR

$(x+2)(x-1) = 0$ $\quad x = -2$

(a) $xy^2 - x^3y = 6$ curve

(b) $(x=1)$

$y^2 - y = 6$

$(y-3)(y+2) = 0$

$y=3 \quad y=-2$

$(1,3) \; (1,-2)$

$\boxed{\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}}$

(a) $\left.\dfrac{dy}{dx}\right|_{\substack{x=1\\y=3}} = \dfrac{9-9}{6-1} = 0 \qquad y=3$

$\left.\dfrac{dy}{dx}\right|_{\substack{x=1\\y=-2}} = \dfrac{-6-4}{-4-1} = \dfrac{-10}{-5} = 2 \qquad \boxed{y+2 = 2(x-1)}$

(c) $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$

$2xy - x^3 = 0$

$x(2y - x^2) = 0$

$\cancel{x=0} \qquad x^2 = 2y$

$y = \dfrac{x^2}{2}$

$xy^2 - x^3y = 6$

$x\left(\dfrac{x^2}{2}\right)^2 - x^3\left(\dfrac{x^2}{2}\right) = 6$

$\dfrac{x^5}{4} - \dfrac{x^5}{2} = 6$

$-\dfrac{x^5}{4} = 6$

$x^5 = -24$

$x = \sqrt[5]{-24}$

≈ -1.89

$f(2)=3$; $f'(2)=4$; $g(2)=5$ $g'(2)=-1$

(3) (a) $h(x)=\dfrac{g(x)}{f(x)}$ $\dfrac{f(2)(g'(2))-g(2)\cdot f'(2)}{[f(2)]^2}$

$h'(2)=\dfrac{3\cdot(-1)-(4)(5)}{3^2}=\dfrac{-23}{9}$

(b) $h(x)=g(x)\cdot f(x)=$
$h'(2)=5\cdot 4+(-1)(3)=17$

(c) $h(x)=g(x)-f(x)$
$h'(2)=-1-4=-5$

(d) $f^{-1}(3)=2$

(4) $f(x)=x\sqrt{2x}+\cos x$ $\quad x^{3/2}\sqrt{2}$

(a) $f'(x)=\frac{3}{2}x^{1/2}\sqrt{2}+(-\sin x)$
$=\dfrac{3\sqrt{2x}}{2}-\sin x$

(b) $f'(\pi)=\dfrac{3\sqrt{2\pi}}{2}-\sin\pi=\dfrac{3\sqrt{2\pi}}{2}$
$f(\pi)=\pi\sqrt{2\pi}-1$
$y-\pi\sqrt{2\pi}+1=\dfrac{3\sqrt{2\pi}}{2}(x-\pi)$

(5) I. T II. F III. T IV. T

(6) $\dfrac{x^2}{25}+\dfrac{y^2}{4}=1$ when $x=2$, $y=\pm\dfrac{2\sqrt{21}}{5}$

$\dfrac{4}{25}+\dfrac{y^2}{4}=1$

$\dfrac{2}{25}x+\dfrac{1}{2}y\dfrac{dy}{dx}=0$ $\quad \dfrac{y^2}{4}=\dfrac{21}{25}$ $\quad y^2=\dfrac{84}{25}$

$y=\pm\dfrac{2\sqrt{21}}{5}$

$\dfrac{dy}{dx}=\left(-\dfrac{2x}{25}\right)\left(\dfrac{2}{y}\right)=-\dfrac{4x}{25y}\Big|_{x=2}$

$\dfrac{-4(2)}{25\left(\pm\frac{2\sqrt{21}}{5}\right)}=\dfrac{-8}{\pm 10\sqrt{21}}=\pm\dfrac{4}{5\sqrt{21}}$

(7) $\frac{d}{dx}\cos^2(x^3) = 2\cos(x^3)(-\sin x^3)\cdot 3x^2$

$= -6x^2\cos x^3 \sin x^3$

(d)

(8)

$2x - 4y = 3$

$y = 0.5x^2$

$m = \frac{1}{2}$

$y' = x$

$x = \frac{1}{2}$

$y = 0.5\left(\frac{1}{4}\right) = \frac{1}{8}$

$\left(\frac{1}{2}, \frac{1}{8}\right)$

(6) $x = 2$

$\frac{x^2}{25} + \frac{y^2}{4} = 1$

10 units wide
4 units high

$\frac{4}{25} + \frac{y^2}{4} = 1$

$\frac{2}{25}x + \frac{1}{2}y\frac{dy}{dx} = 0$

$\frac{2}{y}\left(\frac{y}{2}\frac{dy}{dx}\right) = -\frac{2x}{25}\cdot\frac{2}{y}$

$\frac{y^2}{4} = \frac{21}{25}$

$\frac{dy}{dx} = -\frac{4x}{25y}$

$y^2 = \frac{21\cdot 4}{25}$

$y = \pm\frac{2\sqrt{21}}{5}$

$\left.\frac{dy}{dx}\right|_{\substack{x=2 \\ y=\pm\frac{2\sqrt{21}}{5}}} = \frac{-8}{25\left(\frac{2\sqrt{21}}{5}\right)} = \frac{-8}{10\sqrt{21}}$

$\pm\frac{4}{5\sqrt{21}}$