AP Calculus AB
Indefinite Integrals (U-Sub & a couple algebraic techniques) ★

Name: KEY
Block: ________

1. $\int \frac{e^x}{1+2e^x}\,dx$

$$\boxed{\frac{1}{2}\ln\left(1+2e^x\right)+C}$$

2. $\int \sec^2(2x)\,dx$

$$\boxed{\frac{1}{2}\tan(2x)+C}$$

3. $\int \sec^2(3x)e^{\tan(3x)}\,dx$

$$\boxed{\frac{1}{3}e^{\tan(3x)}+C}$$

4. $\int \frac{x}{2x^2+1}\,dx$

$$\boxed{\frac{1}{4}\ln\left(2x^2+1\right)+C}$$

5. $\int e^x\left(2+e^x\right)^{1/2}\,dx$

$$\boxed{\frac{2}{3}\left(2+e^x\right)^{3/2}+C}$$

6. $\int x^2\cos\left(x^3\right)dx$

$$\boxed{\frac{1}{3}\sin\left(x^3\right)+C}$$

7. $\int \frac{\sec^2 x}{\sqrt{\tan x}}\,dx$

$$\boxed{2\sqrt{\tan x}+C}$$

8. $\int \frac{\tan^{-1}x}{1+x^2}\,dx$

$$\boxed{\frac{1}{2}\left[\tan^{-1}(x)\right]^2+C}$$

9. $\int \csc^2(3x+5)\,dx$

$$\boxed{-\frac{1}{3}\cot(3x+5)+C}$$

10. $\int \frac{x+1}{\left(x^2+2x+7\right)^3}\,dx$

$$\boxed{-\frac{1}{4}\left(x^2+2x+7\right)^{-2}+C}$$

11. $\int \frac{x}{x^2-4}\,dx$

$$\boxed{\frac{1}{2}\ln\left|x^2-4\right|+C}$$

12. $\int x\tan^2\left(x^2\right)dx$

$$\boxed{\frac{1}{2}\tan\left(x^2\right)-\frac{1}{2}x^2+C}$$

13. $\int \cos(3x)e^{\sin(3x)}\,dx$

$$\boxed{\frac{1}{3}e^{\sin(3x)}+C}$$

14. $\int \frac{1}{x\ln(3x)}\,dx$

$$\boxed{\ln\left|\ln(3x)\right|+C}$$

15. $\int \frac{\sin(3x)}{1+\cos(3x)}\,dx$

$$\boxed{-\frac{1}{3}\ln\left|1+\cos(3x)\right|+C}$$

★ 16. $\int \frac{1}{x^2-2x+17}\,dx$

$$\boxed{\frac{1}{4}\tan^{-1}\left(\frac{x-1}{4}\right)+C}$$

★ 17. $\int \frac{1}{\sqrt{1-9x^2}}\,dx$

$$\boxed{\frac{1}{3}\sin^{-1}(3x)+C}$$

18. $\int x\csc\left(3x^2\right)\cot\left(3x^2\right)dx$

$$\boxed{-\frac{1}{6}\csc\left(3x^2\right)+C}$$

19. $\int \frac{1-e^{-x}}{x+e^{-x}}\,dx$

$$\boxed{\ln\left|x+e^{-x}\right|+C}$$

★ 20. $\int \frac{x^2-1}{x^2+1}\,dx$

$$\boxed{x-2\tan^{-1}(x)+C}$$

★ 21. $\int \frac{x^5-35x}{x^2+6}\,dx$

$$\boxed{\frac{1}{4}x^4-3x^2+\frac{1}{2}\ln\left(x^2+6\right)+C}$$

① $\int \frac{e^x}{1+2e^x}\,dx$

Let $u = 1+2e^x$

$du = 2e^x dx \Rightarrow \frac{1}{2}du = e^x dx$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u| + C$

$= \boxed{\frac{1}{2}\ln\left(1+2e^x\right) + C}$

② $\int \sec^2(2x)\,dx$

Let $u = 2x$

$du = 2dx \Rightarrow \frac{1}{2}du = dx$

$= \frac{1}{2}\int \sec^2(u)\,du$

$= \frac{1}{2}\tan(u) + C$

$= \boxed{\frac{1}{2}\tan(2x) + C}$

③ $\int \sec^2(3x)\, e^{\tan(3x)}\,dx$

Let $u = \tan(3x)$

$du = \sec^2(3x)\cdot 3\,dx \Rightarrow \frac{1}{3}du = \sec^2(3x)\,dx$

$= \frac{1}{3}\int e^u\,du$

$= \frac{1}{3}e^u + C$

$\boxed{= \frac{1}{3}e^{\tan(3x)} + C}$

④ $\int \frac{x}{2x^2+1}\,dx$

Let $u = 2x^2+1$

$du = 4x\,dx \Rightarrow \frac{1}{4}du = x\,dx$

$= \frac{1}{4}\int \frac{1}{u}\,du$

$= \frac{1}{4}\ln|u| + C$

$\boxed{= \frac{1}{4}\ln(2x^2+1) + C}$

⑤ $\int e^x(2+e^x)^{1/2}\,dx$

Let $u = 2+e^x$

$du = e^x dx$

$= \int u^{1/2}\,du = \frac{2}{3}u^{3/2} + C$

$\boxed{= \frac{2}{3}(2+e^x)^{3/2} + C}$

⑥ $\int x^2 \cos(x^3)\,dx$

$u = x^3$

$du = 3x^2\,dx \Rightarrow \frac{1}{3}du = x^2\,dx$

$= \frac{1}{3}\int \cos(u)\,du$

$= \frac{1}{3}\cdot \sin(u) + C$

$\boxed{= \frac{1}{3}\sin(x^3) + C}$

⑦ $\int \frac{\sec^2 x}{\sqrt{\tan x}}\,dx$

Let $u = \tan x$

$du = \sec^2 x\,dx$

$= \int \frac{1}{\sqrt{u}}\,du$

$= \int u^{-1/2}\,du$

$= 2u^{1/2} + C$

$\boxed{= 2\sqrt{\tan x} + C}$

⑧ $\int \frac{\tan^{-1} x}{1+x^2}\,dx$

Let $u = \tan^{-1} x$

$du = \frac{1}{1+x^2}\,dx$

$= \int u\,du = \frac{1}{2}u^2 + C$

$\boxed{= \frac{1}{2}\left[\tan^{-1} x\right]^2 + C}$

⑨ $\int \csc^2(3x+5)\,dx$

Let $u = 3x+5$

$du = 3\,dx \Rightarrow \frac{1}{3}du = dx$

$= \frac{1}{3}\int \csc^2(u)\,du$

$= \frac{1}{3}(-\cot(u)) + C$

$\boxed{= -1/3 \cot(3x+5) + C}$

⑩ $\int \frac{x+1}{(x^2+2x+7)^3}\,dx =$

Let $u = x^2+2x+7$

$du = (2x+2)\,dx \Rightarrow du = 2(x+1)\,dx$

$\Rightarrow \frac{1}{2}du = (x+1)\,dx$

$= \frac{1}{2}\int \frac{1}{u^3}\,du$

$= \frac{1}{2}\int u^{-3}\,du$

$= \frac{1}{2}\left(-\frac{1}{2}u^{-2}\right) + C$

$\boxed{= -1/4\,(x^2+2x+7)^{-2} + C}$

(11) $\int \frac{x}{x^2-4}\,dx =$

Let $u = x^2-4$

$du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u| + C$

$\boxed{= \frac{1}{2}\ln\left|x^2-4\right| + C}$

(12) $\int x\tan^2(x^2)\,dx$

$u = x^2$

$du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int \tan^2(u)\,du$

$= \frac{1}{2}\int \left[\sec^2(u) - 1\right]du$

$= \frac{1}{2}\left[\tan(u) - u\right] + C$

$= \frac{1}{2}\tan(u) - \frac{1}{2}u + C$

$\boxed{= 1/2\,\tan(x^2) - \frac{1}{2}x^2 + C}$

(13) $\int \cos(3x)e^{\sin(3x)}\,dx$

Let $u = \sin(3x)$

$du = \cos(3x)\cdot 3\,dx$

$\Rightarrow \frac{1}{3}du = \cos(3x)\,dx$

$= \frac{1}{3}\int e^u\,du$

$= \frac{1}{3}e^u + C$

$\boxed{= \frac{1}{3}e^{\sin(3x)} + C}$

(14) $\int \frac{1}{x\ln(3x)}\,dx$

Let $u = \ln(3x)$

$du = \frac{3}{3x}\,dx = \frac{1}{x}\,dx$

$= \int \frac{1}{u}\,du$

$= \ln|u| + C$

$\boxed{= \ln\left|\ln(3x)\right| + C}$

(15) $\int \frac{\sin(3x)}{1+\cos(3x)}\,dx$

Let $u = 1+\cos(3x)$

$du = -\sin(3x)\cdot 3\,dx \Rightarrow -\frac{1}{3}du = \sin(3x)\,dx$

$= -\frac{1}{3}\int \frac{1}{u}\,du$

$= -\frac{1}{3}\ln|u| + C$

$\boxed{= -\frac{1}{3}\ln\left|1+\cos(3x)\right| + C}$

(16) $\displaystyle\int \frac{1}{x^2-2x+17}\,dx = \int \frac{1}{x^2-2x+\underline{1}+17-\underline{1}}\,dx$

$$= \int \frac{1}{(x-1)^2+16}\,dx$$

$$= \int \frac{1}{16\left[\frac{(x-1)^2}{16}+1\right]}\,dx$$

Let $u = \frac{x-1}{4}$ ... so that $u^2 = \frac{(x-1)^2}{16}$

$du = \frac{1}{4}\,dx \Rightarrow 4\,du = dx$

$$= \frac{4}{16}\int \frac{1}{u^2+1}\,du$$

$$= \frac{1}{4}\tan^{-1}(u) + C$$

$$\boxed{= \frac{1}{4}\tan^{-1}\left(\frac{x-1}{4}\right) + C}$$

(17) $\displaystyle\int \frac{1}{\sqrt{1-9x^2}}\,dx$

Let $u = 3x$

$du = 3\,dx \Rightarrow \frac{1}{3}\,du = dx$

$$= \frac{1}{3}\int \frac{1}{\sqrt{1-u^2}}\,du$$

$$= \frac{1}{3}\sin^{-1}(u) + C$$

$$\boxed{= \frac{1}{3}\sin^{-1}(3x) + C}$$

(18) $\displaystyle\int x\csc(3x^2)\cot(3x^2)\,dx$

Let $u = 3x^2$

$du = 6x\,dx \Rightarrow \frac{1}{6}\,du = x\,dx$

$$= \frac{1}{6}\int \csc(u)\cot(u)\,du$$

$$= \frac{1}{6}\left[-\csc(u)\right] + C$$

$$\boxed{= -\frac{1}{6}\csc(3x^2) + C}$$

(19) $\int \frac{1-e^{-x}}{x+e^{-x}}\,dx$

Let $u = x + e^{-x}$

$du = 1 + \underset{-1}{e^{-x}}\,dx = (1-e^{-x})dx$

$= \int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \boxed{\ln|x+e^{-x}| + C}$

(20) $\int \frac{x^2-1}{x^2+1}\,dx = \int \left(1 + \frac{-2}{x^2+1}\right)dx$

$$\begin{array}{r|l} & 1 \\ x^2+0x+1 & x^2+0x-1 \\ & -(x^2+0x+1) \\ \hline & -2 \end{array}$$

$= \int 1\,dx - 2\int \frac{1}{x^2+1}\,dx$

$\boxed{= x - 2\tan^{-1}(x) + C}$

(21) $\int \frac{x^5-35x}{x^2+6}\,dx = \int \left(x^3 - 6x + \frac{x}{x^2+6}\right)dx$

$$\begin{array}{r|l} & x^3 - 6x \\ x^2+0x+6 & x^5+0x^4+0x^3+0x^2-35x+0 \\ & -(x^5+0x^4+6x^3) \\ \hline & -6x^3+0x^2-35x \\ & -(-6x^3+0x^2-36x) \\ \hline & x \end{array}$$

$= \frac{1}{4}x^4 - 3x^2 + \int \frac{x}{x^2+6}\,dx$

$u = x^2+6$

$du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

$= \frac{1}{4}x^4 - 3x^2 + \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{4}x^4 - 3x^2 + \frac{1}{2}\ln|u| + C$

$\boxed{= \frac{1}{4}x^4 - 3x^2 + \frac{1}{2}\ln(x^2+6) + C}$