16.) pt: $(-3, 8)$

$m = -\frac{1}{9}$

$\boxed{y - 8 = -\frac{1}{9}(x+3)}$

~~First~~ review answers

17.) IVT

1.) $f(x)$ continuous $[0, 4]$? yes

2.) $f(0) = -3$

$f(4) = 5$

3.) $f(a) < k < f(b)$

$-3 < 2 < 5$ ∴ IVT holds.

Find c

$2 = (x-1)^2 - 4$

$6 = (x-1)^2$

$\pm\sqrt{6} = x - 1$

$x = 1 \pm \sqrt{6}$

only $x = 1 + \sqrt{6}$ will work

∴ $\boxed{c = 1 + \sqrt{6}}$

18.) $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

∴ $\lim_{h \to 0} \frac{f(-5+h) - f(-5)}{h} = \lim_{h \to 0} \frac{-7\cancel{h} + 4h^{\cancel{2}} - 2h^{\cancel{3}\,2}}{\cancel{h}} = \lim_{h \to 0} -7 + \cancel{4h}^{0} - \cancel{2h^2}^{0}$

$= \boxed{-7}$

19.) $p(x) = f(x) \cdot g(x)$

$p'(x) = g'(x) \cdot f(x) + f'(x) g(x)$

$p'(4) = (-1)(1) + (\frac{1}{2})(2) = \boxed{0}$

$-1 \quad +1$

20.) $w(x) = g(f(x))$

$w'(x) = f'(x) \cdot g'(f(x))$

$w'(1) = f'(1) \cdot g'(f(1))$

$= -3 \cdot g'(3)$

$= -3 \cdot -1 = \boxed{3}$

21.) $v(x) = \frac{f(x)}{g(x)}$

$v'(x) = \frac{g(x) f'(x) - f(x) g'(x)}{[g(x)]^2}$

$v'(3) = \frac{3 \cdot \frac{1}{2} - \frac{1}{2} \cdot (-1)}{3^2} = \frac{\frac{3}{2} + \frac{1}{2}}{9} = \boxed{\frac{2}{9}}$

22.) $f(x) = x^2 - 5x - x^{\frac{1}{4}}$

$$\boxed{F'(x) = 2x - 5 - \frac{1}{4x^{\frac{3}{4}}}}$$

~~23.) $F(x) = x^4 \cdot \sin(x^3) + \tan^3(6x^5)$~~

~~$F'(x) = 3x^2$~~

~~$f'(x) = \cos(x^3) \cdot 3x^2 \cdot x^4$~~

23.) $f(x) = x^4 \sin(x^3) + [\tan(6x^5)]^3$

$$\boxed{F'(x) = \cos(x^3) \cdot 3x^2 \cdot x^4 + 4x^3 \cdot \sin(x^3) + [3\tan(6x^5)]^2 \cdot \sec^2(6x^5) \cdot 30x^4}$$

24.) $y = \cancel{\cos}(\csc(4x^5))^{\frac{1}{3}}$

$y' = \frac{1}{3}\boxed{(\csc(4x^5))^{-\frac{2}{3}}} \cdot -\csc(4x^5)\cot(4x^5) \cdot 20x^4$

$$\boxed{y' = \frac{-\csc(4x^5)\cot(4x^5) \cdot 20x^4}{3(\csc(4x^5))^{\frac{2}{3}}}}$$

25.) a.) $2y^3 + 6x^2y - 12x^2 + 6y = 1$

$\frac{dy}{dx}6y^{2} + \frac{dy}{dx}6x^2 + 12xy - 24x + 6\frac{dy}{dx} = 0$

~~$\frac{dy}{dx}(6x^2 - 6) = -6y$~~

$\frac{dy}{dx}(6y^2 + 6x^2 + 6) = -12xy + 24x$

$\div 6$

$$\boxed{\frac{dy}{dx} = \frac{-12xy + 24x}{6y^2 + 6x^2 + 6}} \Rightarrow \boxed{\frac{4x - 2xy}{x^2 + y^2 + 1}}$$

~~b.) hor. tangent: $m = 0$~~

~~$4x - 2xy = 0$~~

~~$x(4 - 2y) = 0$~~

~~$x = 0$ or $y = 2$~~

~~$2y^3 + 6y = 1$~~

~~$y =$~~

2.5 b.) hor. tangent: $m=0$

$4x-2xy=0$

$x(4-2y)=0$

$x=0$ or $y=2$

points

$\underline{x=0}$

$2y^3+6y=1$

$2y^3+6y-1=0$

$y=.165$

$(0,.165)$

$\underline{y=2}$

$16+\cancel{12x^2}-\cancel{12x^2}+12=0$

$28=0$

$\boxed{\emptyset}$

slope:
@ $(0,.165)$

$m=\frac{4(0)-2(0)(.165)}{0^2+.165^2+1}=0$

$\underline{\text{tangent line}}$

$\boxed{y=.165}$

c.) ~~$m=-1$~~ ~~pt: (0,0)~~

$m=-1$

pt $(0,0)$

$\underline{\text{Tangent line}}$

$y=-1x$

$\frac{dy}{dx}=\frac{4x-2xy}{x^2+y^2+1}$

$-1=\frac{4x+2x^2}{x^2+\cancel{y^2}+1}$

$-2x^2-1=4x+2x^2$

$4x^2+4x+1=0$

$(2x+1)^2=0$

$\therefore \boxed{x=-\frac{1}{2}}$ $\therefore \boxed{y=\frac{1}{2}}$

26.)

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = 10\ ft^3/min$

$r = \frac{1}{2}h$

$h = 8$

find $\frac{dh}{dt}$?

$\cancel{\frac{dv}{dt} = \frac{2}{3}}$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 \cdot h$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$10 = \frac{1}{4}\pi(8)^2 \cdot \frac{dh}{dt}$

$\therefore \frac{dh}{dt} = .199\ ft/min$

$\boxed{\text{* The height is increasing at a rate of .199 ft/min.}}$

27.) $\lim_{x \to 3^+} f(x) = 6(3) - 9 = 9$

$\lim_{x \to 3^-} f(x) = 3^2 = 9$

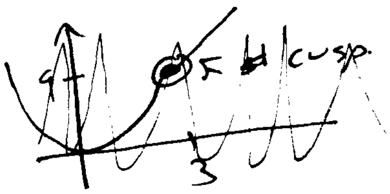
$\therefore$ continuous.

$f'(x) = 2x$

$f'(3) = 6$ same

$f'(x) = 6$

$\therefore$ cont. + diff.



$\therefore$ ~~continuous but not differentiable~~ ~~B~~

$\boxed{E}$

28.) $f'(x) = 3x^2 - 6x$

$0 = 3x^2 - 6x$

$0 = 3x(x-2)$

$x = 0 \quad x = 2$

	x	f(x)
CN's	0	12
	2	8
end pts	-2	-8
	4	28 max

$\boxed{A}$

29.) $f'(x) = 3x^2+6x$

$f''(x) = 6x+6=0$

$x=-1$

p=

pt:

$x=-1$

$y=-1+3+2$

$y=4$

$\boxed{(-1,4)}$

slope

$f'(-1) = 3-6 = -3$

30.) $x^2+y^2=z^2$

$x^2+y^2=z^2 \therefore z=5$

$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}z\frac{dz}{dt}$

~~4(3) + (3)~~

$4\left(3\frac{dy}{dt}\right)+3\left(\frac{dy}{d}\right) = 5(1)$

$12\frac{dy}{dt}+3\frac{dy}{dt} = 5$

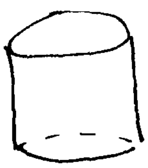
$15\frac{dy}{dt} = 5 \therefore \frac{dy}{dt} = \frac{1}{3}$

$\frac{dx}{dt} = 3\frac{dy}{dt}$

$\therefore \boxed{\frac{dx}{dt} = 1}$

$\boxed{B}$

31.) $SA = 2\pi r^2+2\pi rh$



$V: \cancel{\pi}r^2h = 16\cancel{\pi}$

$r = \frac{4}{\sqrt{h}}$

$SA: 2\pi\frac{16}{h}+2\pi\frac{4}{\sqrt{h}}\cdot h$

$2\pi\frac{16}{h}+8\pi h^{\frac{1}{2}}$

$SA \quad \frac{32\pi}{h}+8\pi h^{\frac{1}{2}}$

$SA' = \boxed{-\frac{32\pi}{h^2}+\frac{4\pi}{h^{\frac{1}{2}}} = 0}$

use calculator

$h=4$

$\boxed{D}$

*32.) $y = x^3 + ax^2 + bx - 4$

$y' = 3x^2 + 2ax + b$

$y'' = 6x + 2a$

$0 = 6(1) + 2a$

$a = -3$

pt $(1, -6)$

$a = -3$

$y = x^3 + ax^2 + bx - 4$

$-6 = (1)^3 + \cancel{2}^{-3}(1)^2 + b(1) - 4$

$-6 = 1 + \cancel{2}^{-3} + b - 4$

$-6 = -6 + b$

$\boxed{b = 0}$

33.) $F\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$f\left(\frac{3\pi}{2}\right) = \sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$

AROC = IROC

$0 = \frac{1}{2}\cos\left(\frac{x}{2}\right)$

$\therefore \cos\left(\frac{x}{2}\right) = 0$

$\frac{x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \pi$, $\cancel{3\pi}$

$\boxed{D}$

34.) $\lim_{h \to 0} \frac{\tan\left(\frac{\pi}{4} + h\right) - \tan\frac{\pi}{4}}{h}$

$f(x) = \tan x$

find $f'\left(\frac{\pi}{4}\right)$

$f'(x) = \sec^2 x$

$\therefore f'\left(\frac{\pi}{4}\right) = \sec^2\left(\frac{\pi}{4}\right)$

$= \frac{2}{\sqrt{2}} = \frac{4}{2} = 2$

$\boxed{E}$

*35.) a.) $x = 1$ $\quad 2(1) - 1^2 = 1$

$x^2 + kx + p = 1$

$1 + k(1) + p = 1$

$\lim_{x \to 1} <$ $\quad k + p = 1$

Differentiability

$2 - 2x = 2x + k$

$4x + k = 2$

$4(1) + k = 2$

$\boxed{k = -2}$

$\therefore \boxed{p = 2}$

36.) a.) $\frac{dw}{dt}$ is negative b/c the rate of change of the weight of paper is decreasing.

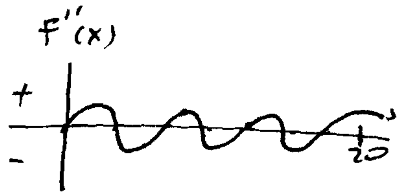
b.) $F(2)=6$ says that after 2 minutes the weight of the paper is 6 pounds.

c.) $F'(2)=-\frac{1}{2}$ says that after 2 minutes the weight of the paper is decreasing at a rate of $\frac{1}{2}$ pound per minute.

37.) $f''(x) = .5 + \cos x - e^{-x}$

From calculator

$[0, 20]$



$\boxed{16}$

6. ∴ $\boxed{C}$

38.) $f'(x) = 24x^2 - 10x - 4 = 0$

$12x^2 - 5x - 2 = 0$

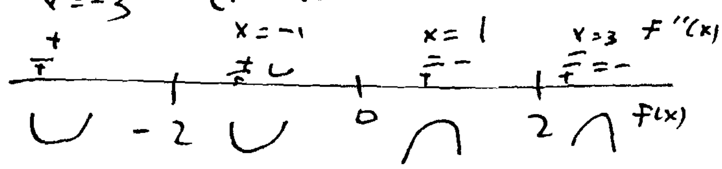
from calculator: $x = -.25$
(zeros) $x = .667$

	x	y
cr's	-.25	3.5625
	.67	1.029396
at EP's	-1	-6 ← min.
	1	2

$\boxed{\text{abs. min @ } x=-1}$ w/

39.) $f''(x) = \frac{(x^2-4)4x - (2x^2-1)2x}{(x^2-4)^2} = \frac{4x^3 - 16x - 4x^3 + 2x}{(x^2-4)^2} = 0$ $x = 0$

$x = \pm 2$



$\frac{-14x}{(x^2-4)^2}$

$\boxed{D}$ $(0,2) \cup (2,\infty)$

40.) $\sqrt[3]{130}$ $\qquad$ $\sqrt[3]{125} = 5$

$F(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$

$F'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$

pt: $(125, 5)$

$m = \frac{1}{3(125)^{\frac{2}{3}}} = \frac{1}{75}$

$y - 5 = \frac{1}{75}(x - 125)$

$y - 5 = \frac{1}{75}(130 - 125)$

$y - 5 = \frac{1}{75}(5)$

$\boxed{y = 5.067}$

41.) a.) $v(t) = 0$

$(t+1)(\sin(\frac{t^2}{2})) = 0$

$t + 1 = 0$ $\qquad$ $\sin(\frac{t^2}{2}) = 0$

$\cancel{t = -1}$ $\qquad$ calculator

outside domain $\qquad$ $t = 2.507$

$t = 3.545$

$v(t)$: $+$ on $(0, 2.507)$, $-$ on $(2.507, 3.545)$, $+$ after 3.545

$0 \nearrow 2.507 \searrow 3.545 \nearrow$

right: $(0, 2.507) \cup (3.545, \infty)$ b/c $v(t) > 0$

test:

b.) $a(t) = v'(t)$

Find $v'(3)$ on calculator $(\frac{d}{dx}y)$

$\boxed{v'(3) = -2.242}$

c.) $v'(3) = -2.242 = a(t)$

$a(t) = -2.242$
$v(t) = -$
$\Big\}$ both negative, $\therefore$ speeding up, but going backwards.

42.) $x(t) = t^3 - 7t^2 + 12t + 4$

a.) $x'(t) = \boxed{3t^2 - 14t + 12 = 0}$

$\boxed{t = 1.131 \quad t = 3.535 \quad b/c \quad x'(t) = 0}$

b.) $x(6) - x(0)$

$40 - 4 = 36$

After 6 seconds the particle ~~to~~ traveled 36 meters.

c.) $x(1.131) - x(0)$ $\quad$ $x(3.535) - x(0)$ $\quad$ $x(6) - x(3.535)$

0 $\quad$ 1.131 $\quad$ 3.535 $\quad$ 6

$= \boxed{49.888\text{m}}$

43.) a.) ~~where $f''(x)$ changes signs.~~
where $f''(x) < 0$

$\boxed{(-4, 0) \cup (8, \infty) \quad b/c \quad f''(x) < 0}$

b.) where $f''(x) = 0$ and changes signs.

$\boxed{x = -4, 0, 8}$

44 a.) where $f'(x) > 0$

$\boxed{(3, 0) \cup (7, \infty) \quad b/c \quad f'(x) > 0}$

b.) where $f'(x) = 0$ and changes from $-$ to $+$

$\boxed{x = -3, 7}$