FTC - Solutions P. 6-7

1) Method 1:

$$\int\left(2+\frac{1}{x^2}\right)dx = \int 2\,dx + \int x^{-2}dx = 2x - \frac{1}{x} + C \qquad y(3) = 2(3) - \frac{1}{(3)} + 5 = \boxed{\frac{32}{3}}$$

$$y(1) = 6 \qquad 6 = 2(1) - \frac{1}{(1)} + C \qquad C = 5 \qquad y(x) = 2x - \frac{1}{x} + 5$$

Method 2:

$$\int_1^3 \left(2+\frac{1}{x^2}\right)dx = y(3) - y(1) \qquad \int_1^3\left(2+\frac{1}{x^2}\right)dx = \left(2x - \frac{1}{x}\right|_1^3$$

$$\left(2(3) - \frac{1}{3}\right) - \left(2(1) - \frac{1}{1}\right) = \frac{14}{3}$$

$$\frac{14}{3} = y(3) - 6$$

$$\boxed{y(3) = \frac{32}{3}}$$

2) $$\int_0^{\pi/4} \cos(2x)\,dx = f\left(\frac{\pi}{4}\right) - f(0)$$

M1: $f(x) = \frac{1}{2}\sin(2x) + C = \frac{1}{2}\sin(2x) + 3$

$f(0) = \frac{1}{2}\sin(2(0)) + C = 3$ $\quad \boxed{f\left(\frac{\pi}{4}\right) = 3.5}$

$C = 3$

M2: $.5 = f\left(\frac{\pi}{4}\right) - 3$

$$\boxed{f\left(\frac{\pi}{4}\right) = 3.5}$$

3) $$\int_0^{24} \frac{1}{75}(600 + 20t - t^2)\,dt = W(24) - W(0)$$

M1: $W(t) = \frac{1}{75}\left(600t + 10t^2 - \frac{1}{3}t^3\right) + C$

$W(0) = \frac{1}{75}(0) + C = 150$

$C = 150$

$W(t) = \frac{1}{75}\left(600t + 10t^2 - \frac{1}{3}t^3\right) + 150$

$\boxed{W(24) = 357.36 \text{ gallons}}$

M2: $207.36 = W(24) - 150$

$\boxed{W(24) = 357.36 \text{ gallons}}$

4) $$\int_0^1 \cos(x^3)\,dx = f(1) - f(0) \qquad \boxed{f(1) = 2.9317}$$

$0.9317 = f(1) - 2$

5) $$\int_2^5 e^{-x^2}\,dx = f(5) - f(2) \qquad \boxed{f(2) = 0.9959}$$

$0.00415 = 1 - f(2)$

6) $$\int_6^7 5\sin(t^2)\,dt = x(7) - x(6)$$

$x(7) = 3.8371$

$-0.1629 = x(7) - 4$

$\boxed{(3.8371, 0)}$

Note: the particle is moving <u>LEFT</u> from time 6 to 7

7) Total increase in bacteria pop. for first three hours:

$$\boxed{\int_0^3 2^t\,dt = 10.099 \text{ million}}$$

Population ~~after~~ at $t=3$ hours:

$$\int_0^3 2^t\,dt = F(3)-F(0)$$

$$10.099 = F(3)-4$$

$$\boxed{F(3) = 14.099 \text{ million bacteria}}$$

8) $$\int_0^3 \frac{t}{1+t^2}\,dt = s(3)-s(0)$$

$$1.1513 = s(3)-5$$

$$\boxed{s(3) = 6.1513}$$

9) $$\int_1^4 f'(x)\,dx = f(4)-f(1)$$

$$6.2 = f(4)-3$$

$$\boxed{f(4) = 9.2}$$

10) $\int_{-4}^4 f'(x)\,dx$ is the area of a semi-circle with $r=4$

$$A = \frac{\pi r^2}{2} = \frac{\pi(4)^2}{2} = 8\pi$$

$$\int_{-4}^4 f'(x)\,dx = f(4)-f(-4)$$

$$8\pi = 7-f(-4)$$

$$\boxed{f(-4) = 7-8\pi}$$

11) $\int_{-2}^1 f'(x)\,dx = 4.5$ (triangle)

$\int_1^4 f'(x)\,dx = -3$ (triangle under x-axis)

$\int_4^8 f'(x)\,dx = 2\pi$ (semi-circle w/ $r=2$)

a) $$\int_{-2}^1 f'(x)\,dx = f(1)-f(-2)$$

$$4.5 = f(1)-5$$

$$\boxed{f(1) = 9.5}$$

b) $$\int_1^4 f'(x)\,dx = f(4)-f(1)$$

$$-3 = f(4)-9.5$$

$$\boxed{f(4) = 6.5}$$

c) $$\int_4^8 f'(x)\,dx = f(8)-f(4)$$

$$2\pi = f(8)-6.5$$

$$\boxed{f(8) = 6.5+2\pi}$$

12) $\int_3^{3.1} \frac{1+e^x}{x^2}\,dx = f(3.1) - f(3)$ $\boxed{f(3.1) \approx 6.2378}$

$0.2378 = f(3.1) - 6$

13) $\int_1^4 \frac{x^2}{1+x^5}\,dx = f(4) - f(1)$ $f(4) = 5.3757$ $\boxed{D}$

$0.3757 = f(4) - 5$

FTC Solutions 8-10

1) $\int_1^4 f'(x)\,dx = f(4) - f(1)$ $\boxed{f(4) = 29}$
$17 = f(4) - 12$

2) $\int_2^5 (2f(x) + 3)\,dx = 17$ $x\big|_2^5 = 5-2 = 3$

$\int_2^5 2f(x)\,dx + \int_2^5 3\,dx = 2\int_2^5 f(x)\,dx + 3\underbrace{\int_2^5 dx} = 17$

$2\int_2^5 f(x)\,dx + 3(3) = 17 \Rightarrow 2\int_2^5 f(x)\,dx = 8 \Rightarrow$ $\boxed{\int_2^5 f(x)\,dx = 4}$

3) $|A| = 1.5$ $A = -1.5$ (under x-axis)

a) $\int_0^6 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^6 f(x)\,dx$

$3.5 = -1.5 + \int_2^6 f(x)\,dx$

$\boxed{\int_2^6 f(x)\,dx = 5}$ ← area $|B|$

b) $\int_0^6 |f(x)|\,dx = |A| + |B|$

$\int_0^6 |f(x)|\,dx = 1.5 + 5$

$\boxed{\int_0^6 |f(x)|\,dx = 6.5}$

4) $\int_0^{30} \overset{f'}{\text{rate}}\,dt$: Trapezoid estimation: $\underbrace{\frac{6}{2}(-50+-30) + \frac{6}{2}(-30+0) + \frac{6}{2}(0+110) + \frac{6}{2}(110+145) + \frac{6}{2}(145+100)} = 1500$ liters

$f(0) = 12000$ $\int_0^{30} f'(t)\,dt = f(30) - f(0)$ $\boxed{f(30) = 13500 \text{ liters}}$

$1500 = f(30) - 12000$

5) $\int_0^{10} -7e^{-0.3t}\,dt = R(10) - R(0)$ $\boxed{R(10) = 67.83\ ^\circ C}$

$-22.17 = R(10) - 90$

6) $-\int_0^{60} 5-5e^{-.12t}\,dt = F(60)-F(0)$ $\boxed{F(60) = 741.636 \text{ liters}}$

$-258.364 = F(60) - 1000$ (Note: (−) because the water is being pumped OUT)

7) $\int_0^6 f'(x)dx = 2(18)+2(23)+2(25) = 132$ (Right Riemann Sum)

$\cancel{\int_0^6 f(x)dx}$ $\int_0^2 f'(x)dx = f(2)-f(0)$ $\int_2^4 f'(x)dx = f(4)-f(2)$

$2(18) = f(2)-100$ $2(23) = f(4)-136$

$\boxed{f(2)=136}$ $\boxed{f(4)=182}$

$\int_4^6 f'(x)dx = f(6)-f(4)$ SO... $\int_0^6 f'(x)\,dx = f(6)-f(0)$ $f(6)=232$

$2(25) = f(6)-182$ $132 = f(6)-100$ as we know

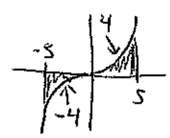
$\boxed{f(6)=232}$

8) a) $\int_0^5 (f(x)+2)\,dx = \int_0^5 f(x)dx + 2\int_0^5 dx = 4+2(5) = \boxed{14}$

b) $\int_{-2}^3 f(x+2)\,dx = \int_0^5 f(x)\,dx = \boxed{4}$ horizontal shift 2 units

c) $\int_{-5}^5 f(x)\,dx$ (f is even) $= 2\int_0^5 f(x)dx = \boxed{8}$

d) $\int_{-5}^5 f(x)dx$ (f is odd) $= \boxed{0}$



$-4+4=0$

9) $P(0)=2$ $\int_0^1 P'(t)dt = P(1)-P(0)$ $\boxed{P(1)=1}$ $\int_1^2 P'(t)dt = P(2)-P(1)$ $\boxed{P(2)=0}$

$-1 = P(1)-2$ $-1 = P(2)-1$

$\int_2^3 P'(t)dt = P(3)-P(2)$ $\boxed{P(3)=-0.5}$ $\int_3^4 P'(t)dt = P(4)-P(3)$ $\boxed{P(4)=0}$

$-0.5 = P(3)-0$ $0.5 = P(4)-(-0.5)$

$\int_4^5 P'(t)dt = P(5)-P(4)$ $\boxed{P(5)=1}$

$1 = P(5)-0$

10) $\int_0^2 F'(x)\,dx = F(2) - F(0)$ $\boxed{F(0) = 1}$ $\boxed{F(2) = 3}$ $\boxed{F(6) = -4}$ $\boxed{F(8) = 0}$

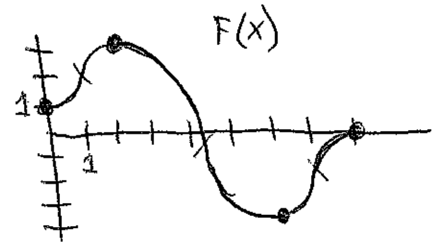
$2 = 3 - F(0)$

$\int_2^6 F'(x)\,dx = F(6) - F(2)$

$-7 = F(6) - 3$

$\int_6^8 F'(x)\,dx = F(8) - F(6)$

$4 = F(8) - (-4)$



\ or / represents inflection points

11) $\boxed{G(0) = 5}$ $\boxed{G(2) = 21}$ $\boxed{G(4) = 13}$ $\boxed{G(5) = 15}$

$\int_0^2 g(t)\,dt = G(2) - G(0)$

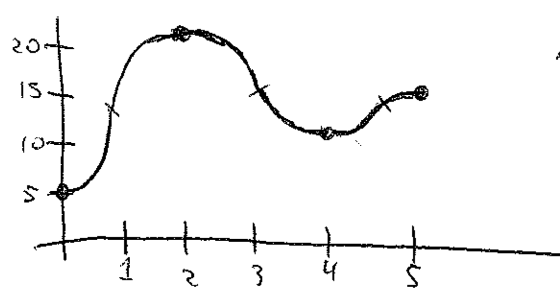
$16 = G(2) - 5$

$\int_2^4 g(t)\,dt = G(4) - G(2)$

$-8 = G(4) - 21$

$\int_4^5 g(t)\,dt = G(5) - G(4)$

$2 = G(5) - 13$



/ or \ represents inflection points

12) $\int_0^1 e^{-x^2}\,dx = F(1) - F(0)$ $\boxed{F(1) = 2.7468}$

$0.7468 = F(1) - 2$

13) ~~$\int_{1/2}^5 f(x)\,dx = \int_{1/2}^1 2\,dx + \int_1^5 2x\,dx = 1 + 24 = \boxed{25}$~~

13) $\int_{1/2}^5 f(x)\,dx = \int_{1/2}^1 2x\,dx + \int_1^5 2\,dx = 0.75 + 8 = \boxed{8.75}$

14) a) $\int_0^{12} T'(x)\,dx = T(12) - T(0) = 93 - 105 = \boxed{-12^\circ F}$ The soup has cooled $12^\circ F$ in the first 12 minutes.

b) $\frac{1}{8-5}\int_5^8 T'(x)\,dx = \left(\frac{1}{3}\right)\left(T(8) - T(5)\right) = \left(\frac{1}{3}\right)(97 - 99) = \boxed{\frac{-2}{3}\ \frac{^\circ F}{min}}$

The soup is cooling at an average of $\frac{2}{3}\ \frac{^\circ F}{min}$ from the 5^{th} minute until the 8^{th} minute.

15) a) $\int_{-2}^{1} f'(x)\,dx = f(1) - f(-2)$ $\qquad \int_{-2}^{0} f'(x)\,dx = -4 \qquad \int_{0}^{1} f'(x)\,dx = 1$

$-3 = 4 - f(-2)$

$\boxed{f(-2) = 7}$

b) $\int_{1}^{5} f'(x)\,dx = f(5) - f(1)$ $\qquad$ semi-circle w/ $r = 2$ $\quad A = \frac{\pi(2)^2}{2} = 2\pi$

$2\pi = f(5) - 4$

$\boxed{f(5) = 4 + 2\pi}$

16) If $g'(x) = f(x)$ ~~and~~ then $\int_{2}^{3} f(x)\,dx = \int_{2}^{3} g'(x)\,dx = \boxed{g(3) - g(2)}$ $\boxed{C}$

17) $\int_{1}^{3} \sqrt{x^3+2}\,dx = g(3) - g(1)$ $\qquad g(1) = \underline{-1.585}$ $\qquad \boxed{B}$

$6.5846 = 5 - g(1)$

18) $\int_{1}^{3} f(x)\,dx = \int_{1}^{3} F'(x)\,dx \cancel{= F(3) - F(1)}$ $\quad \int_{0}^{1} F'(x)\,dx + \int_{1}^{3} F'(x)\,dx = \int_{0}^{3} F'(x)\,dx = F(3) - F(0)$

$\cancel{4.3 - 1}$ $\qquad 2 \quad + \quad 2.3 \quad = \quad \underline{4.3}$

$\boxed{D}$