

Growth and Decay

1) $P(t)$ = Pie temperature

$$\frac{dP}{dt} = k(P-70)$$

$$\int \frac{1}{P-70} dP = \int k dt$$

$$\ln(P-70) = kt + C$$

$$P-70 = Ce^{kt}$$

$$P(t) = Ce^{kt} + 70$$

(time, temp)

$$(0, 450)$$

$$(30, 200)$$

Point 1
 $450 = Ce^{k(0)} + 70$

$$C = 450$$

~~$$P(t) = 450e^{kt}$$~~

$$C = 380$$

$$P(t) = 380e^{kt} + 70$$

Point 2

$$200 = 380e^{k(30)} + 70$$

$$130 = 380e^{k(30)}$$

$$\frac{13}{38} = e^{k(30)}$$

$$30k = \ln \frac{13}{38}$$

$$k = \frac{1}{30} \ln \left(\frac{13}{38} \right)$$

$$k = -0.035755$$

(Decay)

Question: $100 = 380e^{-0.035755t} + 70$

$$30 = 380e^{-0.035755t}$$

$$\frac{3}{38} = e^{-0.035755t}$$

$$-0.035755t = \ln \frac{3}{38}$$

$$t = 71.01 \text{ mins}$$

2) $\frac{dP(t)}{dt} = k\sqrt{P}$

$$P(0) = 2500$$

$$P(5) = 3600$$

$$\int \frac{1}{\sqrt{P}} dP(t) = \int k dt$$

$$2P^{1/2} = kt + C$$

$$P^{1/2} = \frac{kt}{2} + C$$

$$P(t) = \left(\frac{kt}{2} + C \right)^2$$

Point 1
 $2500 = \left(\frac{k(0)}{2} + C \right)^2$

$$50 = C$$

$$P(t) = \left(\frac{kt}{2} + 50 \right)^2$$

$$P(t) = (2t + 50)^2$$

$$P(5) = 3600$$

Point 2
 $3600 = \left(\frac{5k}{2} + 50 \right)^2$

$$60 = \frac{5k}{2} + 50$$

$$10 = \frac{5k}{2}$$

$$k = 4$$

(Growth)

3) $D(t)$ = depth of water in inches

$$\frac{dD}{dt} = \sqrt{D} \cdot k \quad (0, 36)$$

$$(1, 35)$$

Point 1

$$36 = \left(\frac{k(0)}{2} + C\right)^2$$

$$6 = C$$

$$D(t) = \left(\frac{kt}{2} + 6\right)^2$$

$$D(t) = \left(\frac{-0.16784t}{2} + 6\right)^2$$

Point 2

$$35 = \left(\frac{k}{2} + 6\right)^2$$

$$\sqrt{35} = \frac{k}{2} + 6$$

$$k = 2(\sqrt{35} - 6)$$

$$k = -0.16784$$

(decay)

$$D(t) = (-0.08392t + 6)^2$$

$$0 = (-0.08392t + 6)^2$$

~~$$t = 71.5$$~~

$$t = 71.496 \text{ hrs}$$

4) $\frac{dy}{dt} = k(50-y)$

$$y(0) = 0$$

Point 1
 $0 = 50 - Ce^{-k(0)}$

$$C = 50$$

$$y = 50 - 50e^{-kt}$$

$$\int \frac{1}{50-y} dy = \int k dt$$

~~$$y(30) = 20$$~~ hrs

$$y\left(\frac{1}{2}\right) = 20$$

Point 2
 $20 = 50 - 50e^{-k(\frac{1}{2})}$

$$-30 = -50e^{-k(\frac{1}{2})}$$

$$\frac{3}{5} = e^{-k(\frac{1}{2})}$$

$$-\frac{1}{2}k = \ln \frac{3}{5}$$

$$k = -2 \ln \frac{3}{5}$$

$$k = 1.021651$$

(growth)

$$-\ln(50-y) = kt + C$$

$$\ln(50-y) = -kt + C$$

$$50-y = Ce^{-kt}$$

$$y = 50 - Ce^{-kt}$$

$$y(t) = 50 - 50e^{-1.021651t}$$

a) $y(2) = 50 - 50e^{-1.021651(2)}$

$$y(2) = 43.52 \text{ verbs}$$

b) 1 verb left = 49 memorized verbs

$$49 = 50 - 50e^{-1.021651t}$$

$$-1 = -50e^{-1.021651t}$$

$$\frac{1}{50} = e^{-1.021651t}$$

$$t = \frac{\ln \frac{1}{50}}{-1.021651} = 3.829 \text{ hrs}$$