Inverse Trig Functions and Integration

Add these to your Formula List:

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right)+C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\left(\frac{|u|}{a}\right)+C$$

Ex. $\int \frac{dx}{\sqrt{4-x^2}} = \arcsin\left(\frac{x}{2}\right)+C$

Ex. $\int \frac{dx}{\sqrt{4-25x^2}} = \frac{1}{5}\int \frac{5\,dx}{\sqrt{4-(5x)^2}} = \frac{1}{5}\arcsin\left(\frac{5x}{2}\right)+C$

$u = 5x$

$du = 5\,dx$

$\frac{1}{5}\int \frac{du}{\sqrt{2^2-u^2}}$ alternate approach

Ex. $\int_{\sqrt{3}}^{3} \frac{1}{9+x^2}\,dx = \left[\frac{1}{3}\arctan\left(\frac{x}{3}\right)\right]_{\sqrt{3}}^{3} = \frac{1}{3}\left[\arctan 1 - \arctan\frac{\sqrt{3}}{3}\right]$

$\frac{1}{3}\left(\pi/4 - \pi/6\right) = \frac{1}{3}\left(\frac{3\pi}{12} - \frac{2\pi}{12}\right) = \boxed{\pi/36}$

Ex. $\int \frac{dx}{x^2-4x+7} = \int \frac{dx}{(x^2-4x+4)+3} = \int \frac{dx}{(\sqrt{3})^2+(x-2)^2} = \frac{1}{\sqrt{3}}\arctan\left(\frac{x-2}{\sqrt{3}}\right)+C$

or $\frac{\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}(x-2)}{3}\right)+C$

Ex. $\int \frac{1}{\sqrt{-x^2+6x}}\,dx = \int \frac{1}{\sqrt{9-x^2+6x-9}}\,dx = \int \frac{1}{\sqrt{9-(x-3)^2}}\,dx = \arcsin\left(\frac{x-3}{3}\right)+C$

Ex. $\int \frac{x+2}{\sqrt{4-x^2}}\,dx =$ $\int \frac{x}{\sqrt{4-x^2}}\,dx + \int \frac{2}{\sqrt{4-x^2}}\,dx$

$u = 4-x^2$ $\quad -\frac{1}{2}\int u^{-1/2}\,du$

$du = -2x\,dx$

$-\frac{1}{2}\,du = x\,dx$ $\quad -\sqrt{4-x^2} + 2\arcsin\left(\frac{x}{2}\right) + C$

Ex. $\int \frac{2x+5}{x^2+8x+25}\,dx =$ $\int \frac{2x+5}{(x+4)^2+9}\,dx = \int \frac{2x+8}{x^2+8x+25}\,dx - \int \frac{3}{(x+4)^2+9}\,dx$

$\ln|x^2+8x+25| - \frac{3}{3}\arctan\left(\frac{x+4}{3}\right) + C$

Ex. $\int_0^{\sqrt{2}/2} \frac{\arccos x}{\sqrt{1-x^2}}\,dx =$ $-\int u\,du = -\frac{u^2}{2}$

$u = \arccos x$

$du = -\frac{1}{\sqrt{1-x^2}}\,dx$

$= -\left[\frac{(\arccos x)^2}{2}\right]_0^{\sqrt{2}/2}$

$= -\left(\frac{(\pi/4)^2}{2} - \frac{(\pi/2)^2}{2}\right)$

$= -\left(\pi/32 - \pi/8\right)$

$= \boxed{3\pi/32}$

Homework: P. 385: 1 – 41 odd, 47 – 63 odd & AP Review 14