Integration resulting in Inverse Trig functions

Name Key
Period ____________

Evaluate each integral. Work problems on notebook paper.

1) $\int \frac{1}{1+x^2}dx = \frac{1}{1}\arctan\left(\frac{x}{1}\right) + C = \boxed{\arctan x + C}$

2) $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}dx = \left[\arcsin\left(\frac{x}{1}\right)\right]_0^{1/2} = (\arcsin 1/2 - \arcsin 0) = \pi/6 - 0 = \boxed{\pi/6}$

3) $\int \frac{1}{\sqrt{1-4x^2}}dx = \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \arcsin \frac{u}{1} + C = \boxed{\arcsin(2x) + C}$

$u = 2x \qquad a = 1$

$du = 2dx$

$\frac{1}{2}du = dx$

4) $\int_{\sqrt{2}}^{2} \frac{1}{x\sqrt{x^2-1}}dx = \left[\frac{1}{1}\operatorname{arcsec}\frac{|x|}{1}\right]_{\sqrt{2}}^{2} = (\operatorname{arcsec} 2 - \operatorname{arcsec}\sqrt{2})$

$= \pi/3 - \pi/4 = \frac{4\pi}{12} - \frac{3\pi}{12} = \boxed{\frac{\pi}{12}}$

5) $\int_{-1}^{1} \frac{1}{1+x^2}dx = \left[\frac{1}{1}\arctan\left(\frac{x}{1}\right)\right]_{-1}^{1} = (\arctan 1 - \arctan(-1))$

$\pi/4 - (-\pi/4) = \boxed{\pi/2}$

6) $\int_{\frac{1}{\sqrt{3}}}^{1} \frac{1}{8x\sqrt{4x^2-1}}dx = \frac{1}{2}\int_{\frac{2}{\sqrt{3}}}^{2} \frac{1}{4(u)\sqrt{(u)^2-1}}du = \frac{1}{8}\left[\frac{1}{1}\operatorname{arcsec}\frac{|u|}{1}\right]_{\frac{2}{\sqrt{3}}}^{2}$

$\frac{1}{8}\left(\operatorname{arcsec} 2 - \operatorname{arcsec}\frac{2\sqrt{3}}{3}\right)$

$\frac{1}{8}\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \frac{1}{8}\left(\frac{2\pi}{6} - \frac{\pi}{6}\right) = \frac{1}{8}(\pi/6) = \boxed{\frac{\pi}{48}}$

$u = 2x \qquad a = 1$

$du = 2dx$

$\frac{1}{2}du = dx$

7) $\int \frac{1}{1+9x^2}dx = \frac{1}{3}\int \frac{1}{1+u^2}du = \frac{1}{3}\left(\frac{1}{1}\arctan\frac{u}{1}\right)+C = \boxed{\frac{1}{3}\arctan(3x)+C}$

$u = 3x$ $\quad a = 1$

$du = 3dx$

$\frac{1}{3}du = dx$

8) $\int \frac{x}{\sqrt{1-x^4}}dx = \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}du = \frac{1}{2}\arcsin\frac{u}{1}+C = \boxed{\frac{1}{2}\arcsin(x^2)+C}$

$u = x^2$ $\quad a = 1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

9) $\int \frac{e^x}{1+e^{2x}}dx = \int \frac{1}{1+u^2}du = \frac{1}{1}\arctan\frac{u}{1}+C = \boxed{\arctan(e^x)+C}$

$u = e^x$ $\quad a = 1$

$du = e^x dx$

10) $\int \frac{x^2}{1+x^6}dx = \frac{1}{3}\int \frac{1}{1+u^2}du = \frac{1}{3}\cdot\frac{1}{1}\arctan\frac{u}{1}+C = \boxed{\frac{1}{3}\arctan(x^3)+C}$

$u = x^3$ $\quad a = 1$

$du = 3x^2dx$

$\frac{1}{3}du = x^2dx$

11) $\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}dx = 2\int \frac{1}{\sqrt{1-u^2}}du = 2\arcsin\left(\frac{u}{1}\right)+C = \boxed{2\arcsin\left(\frac{x}{2}\right)+C}$

$a = 1$

$u = \frac{x}{2}$

$du = \frac{1}{2}dx$ $\quad 2du = dx$

12) $\int \frac{1}{1+\frac{x^2}{9}}dx = 3\int \frac{1}{1+u^2}du = 3\left(\frac{1}{1}\right)\arctan\frac{u}{1}+C = \boxed{3\arctan\left(\frac{x}{3}\right)+C}$

$a = 1$

$u = \frac{x}{3}$

$du = \frac{1}{3}dx$ $\quad 3du = dx$

Answers:

1. $\arctan x + c$ 2. $\frac{\pi}{6}$ 3. $\frac{1}{2}\arcsin 2x + c$ 4. $\frac{\pi}{12}$ 5. $\frac{\pi}{2}$ 6. $\frac{\pi}{48}$ 7. $\frac{1}{3}\arctan 3x + c$

8. $\frac{1}{2}\arcsin x^2 + c$ 9. $\arctan e^x + c$ 10. $\frac{1}{3}\arctan(x^3)+c$ 11. $2\arcsin\frac{x}{2}+c$ 12. $3\arctan\frac{x}{3}+c$