

Inverse Trigonometric Function Review

Basic Trig functions

$$\cos \frac{\pi}{3} = \frac{1}{2} \quad \sin \frac{7\pi}{4} = -\frac{1}{\sqrt{2}} \quad \tan 120^\circ = \sqrt{3} \quad \csc \frac{\pi}{2} = 1$$

Inverse trig functions work the following way

$$\tan \theta = -1 \quad \theta = \quad \cos \theta = -\frac{1}{2} \quad \theta = \quad \csc \theta = 2 \quad \theta =$$

each of the above equations could be re-written as the following

$$\tan^{-1}(-1) = -\frac{\pi}{4} \quad \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3} \quad \text{arc csc}(2) = \frac{\pi}{6}$$

Because we want these relations to be functions we need to restrict the range that each of the following inverse trig functions have.

$$\cos^{-1}x = \text{Range } [0, \pi]$$

$$\sin^{-1}x = \text{Range } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\sec^{-1}x = \text{Range } [0, \pi]$$

$$\csc^{-1}x = \text{Range } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\cot^{-1}x = \text{Range } [0, \pi]$$

$$\tan^{-1}x = \text{Range } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

When we look at the graphs of these functions we need to make sure that the inverse of each trig function passes the vertical line test.

Answer the following examples.

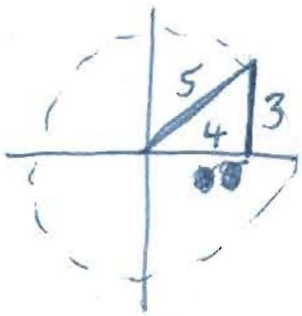
$$\begin{array}{ccccc} \sin^{-1}\left(-\frac{1}{2}\right) & \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) & \tan^{-1}(-\sqrt{3}) & \sec^{-1}(-1) & \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) \\ -\frac{\pi}{6} & \frac{\pi}{6} & -\frac{\pi}{3} & \pi & \frac{3\pi}{4} \end{array}$$

We can also work problems using right triangles.

$$\tan(\cos^{-1}(\frac{4}{5})) = \frac{3}{4}$$

$$\cos(\sin^{-1}(\frac{1}{3})) = \frac{2\sqrt{2}}{3}$$

$$\csc(\cos^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$



$$\begin{aligned}\cos \theta &= \frac{x}{r} \\ \sin \theta &= \frac{y}{r} \\ \tan \theta &= \frac{y}{x}\end{aligned}$$

