1 — 27-12

Pg 358 # 85 - 105 odds

5.4 Exponential fcns: Integration

85-97 Find indefinite integral.

(85) $\int e^{5x}(5)\,dx$ Let $u = 5x$ $du = 5\,dx$

$\int e^u\,du = e^u + C = \boxed{e^{5x} + C}$

(87) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ Let $u = \sqrt{x}$ $du \rightarrow \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}\,dx$

$= 2\int e^{\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\right)dx = 2\int e^u\,du = 2e^u + C = \boxed{2e^{\sqrt{x}} + C}$

(89) $\int \frac{e^{-x}}{1+e^{-x}}\,dx$ Let $u = 1+e^{-x}$ $du = -e^{-x}\,dx$

$= \int \frac{1}{u}\,du = -[\ln|u|] + C = \boxed{-\ln(1+e^{-x}) + C}$ but

$1+e^{-x} = 1 + \frac{1}{e^x} = \frac{e^x+1}{e^x}$ so

$-\ln(1+e^{-x}) + C = \ln(1+e^{-x})^{-1}$

$= \ln\left(\frac{e^x+1}{e^x}\right)^{-1} + C = \ln\frac{e^x}{e^x+1} + C$

$= \ln e^x - \ln(e^x+1) + C = \boxed{x - \ln(e^x+1) + C}$

(91) $\int e^x\sqrt{1-e^x}\,dx$ Let $u = 1-e^x$ $du = -e^x\,dx$

$-\int u^{\frac{1}{2}}\,du = -\left[\frac{2}{3}u^{\frac{3}{2}}\right] + C = \boxed{-\frac{2}{3}(1-e^x)^{\frac{3}{2}} + C}$

(93) $\int \frac{e^x+e^{-x}}{e^x-e^{-x}}\,dx$ Let $u = e^x - e^{-x}$

$\int \frac{1}{u}\,du = \ln|u| + C$ $du = (e^x+e^{-x})\,dx$

$= \boxed{\ln|e^x - e^{-x}| + C}$

(95) $\int \frac{5-e^{x}}{e^{2x}}\,dx = \int \left(5e^{-2x} - e^{-x}\right) dx$

$= \int 5e^{-2x}\,dx - \int e^{-x}\,dx =$

Let $u = -2x$

$du = -2dx$

$dx = -\frac{2}{du}$

$-\frac{1}{2}\int 5e^{-2x}(-2dx) - \int e^{-x}\,dx = -\frac{5}{2}\int e^{u}\,du - \int e^{-x}\,dx$

$= -\frac{5}{2}e^{u} - (-e^{-x}) + c$

$= \boxed{-\frac{5}{2}e^{-2x} + e^{-x} + C}$

(97) $\int e^{-x}\tan(e^{-x})\,dx$ Let $u = e^{-x}$ $du = -e^{-x}\,dx$

$-\int \tan u\,du = -\left[-\ln|\cos u|\right] + C$

$\int \tan u\,du = -\ln|\cos u|$

$= \boxed{+\ln|\cos(e^{-x})| + C}$

(99) $\int_0^1 e^{-2x}\,dx$ Let $u = -2x$ $du = -2dx$

$-\frac{1}{2}\int_0^1 e^{-2x}\cdot -2dx = -\frac{1}{2}\int_0^{-2} e^{u}\,du = \frac{1}{2}\int_{-2}^{0} e^{u}\,du$

$= \frac{1}{2}\left[e^{u}\right]_{-2}^{0} = \frac{1}{2}\left(e^{0} - e^{-2}\right) = \boxed{\frac{1}{2}\left(1 - e^{-2}\right)}$

$= \frac{1}{2}\left(1 - \frac{1}{e^{2}}\right) = \frac{1}{2}\left(\frac{e^{2}-1}{e^{2}}\right) = \boxed{\frac{e^{2}-1}{2e^{2}}}$

(101) $\int_0^1 xe^{-x^2}\,dx$ Let $u=-x^2$ $du=-2x\,dx$

$-\frac{1}{2}\int_0^1 e^{-x^2}\cdot -2x\,dx = -\frac{1}{2}\int_0^{-1} e^u\,du = +\frac{1}{2}\left[e^u\right]_{-1}^{0}$

$= \frac{1}{2}(e^0 - e^{-1}) = \frac{1}{2}\left(1-\frac{1}{e}\right) = \frac{1}{2}\left(\frac{e-1}{e}\right)$

$= \boxed{\frac{e-1}{2e}}$

(103) $\int_1^3 \frac{e^{3/x}}{x^2}\,dx$ Let $u=\frac{3}{x}$ $du=-\frac{3}{x^2}\,dx$

$-\frac{1}{3}\int_1^3 e^{3/x}\cdot\left(-\frac{3}{x^2}\right)dx = -\frac{1}{3}\int_3^1 e^u\,du = \frac{1}{3}\int_1^3 e^u\,du$

$= \frac{1}{3}\left[e^u\right]_1^3 = \frac{1}{3}\left[e^3 - e^1\right] = \boxed{\frac{e^3-e}{3} \text{ or } \frac{e}{3}(e^2-1)}$

(105) $\int_0^{\frac{\pi}{2}} e^{\sin\pi x}\cos\pi x\,dx$ Let $u=\sin\pi x$

$du = \pi\cos\pi x\,dx$

$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{\sin\pi x}\cdot\pi\cos\pi x\,dx$

$= \frac{1}{\pi}\int e^u\,du = \frac{1}{\pi}\left[e^u\right] = \frac{1}{\pi}\left[e^{\sin\pi x}\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{\pi}\left[e^{\sin\frac{\pi^2}{2}} - e^{\sin\pi(0)}\right]$

$= \frac{1}{\pi}\left[e^{\sin\frac{\pi^2}{2}} - e^0\right] = \boxed{\frac{1}{\pi}\left[e^{\sin\left(\frac{\pi^2}{2}\right)} - 1\right]}$