

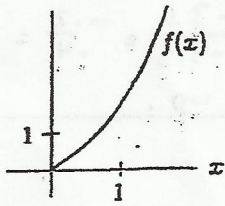
CALCULUS
WORKSHEET ON f, f', f''

Key

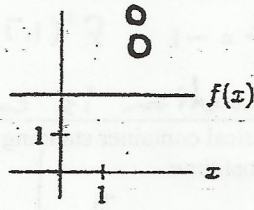
For problems 1 – 6, give the signs of the first and second derivatives for each of the following functions.

1.

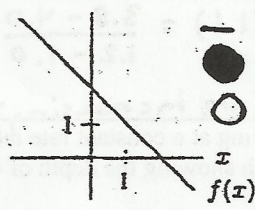
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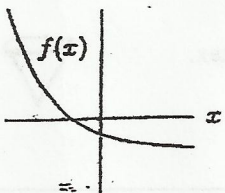


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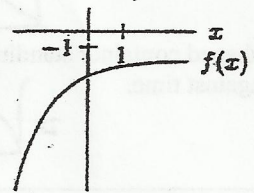
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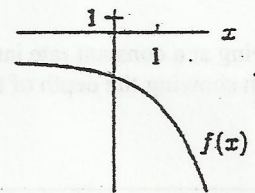
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6.

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7. Let $P(t)$ represent the price of a share of stock of a corporation at time t . What does each of the following statements tell us about the signs of the first and second derivatives of $P(t)$?

(a) "The price of the stock is rising faster and faster."

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(b) "The price of the stock is close to bottoming out."

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8. Given the following data:

x	0	0.2	0.4	0.6	0.8	1.0
$f(x)$	3.7	3.5	3.5	3.9	4.0	3.9

(a) Estimate $f'(0.6)$ and $f'(0.5)$. $f'(.6) \approx \frac{4-3.5}{.8-.4} \approx \frac{.5}{.4} \approx 1.25$ $f'(.5) \approx \frac{3.9-3.5}{.6-.4} \approx \frac{.4}{.2} \approx 2$

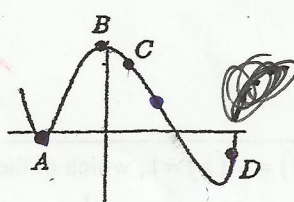
(b) Estimate $f''(0.6)$. $f''(.6) \approx \frac{1.25-2}{.6-.5} \approx \frac{-.75}{.1} \approx -7.5$

x	1.5	1.6
$f(x)$	2	1.25

(c) Where do you think the maximum and minimum values of f occur in the interval $0 \leq x \leq 1$?

Min at .3 max at .8

9. The graph of a function $f(x)$ is shown in the figure below. On the table, indicate whether f, f', f'' at each of the marked points is positive, negative, or zero.



Point	f	f'	f''
A	0	-	+
B	+	0	-
C	+	-	-
D	-	+	+

TURN-->>>

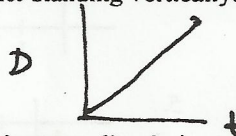
10. Do the values for the function $y = k(x)$ in the table below suggest that the graph of $k(x)$ is concave up or concave down for $1 \leq x \leq 3.3$? Write a sentence in support of your conclusion.

x	1.0	1.2	1.5	1.9	2.5	3.3
$k(x)$	4.0	3.8	3.6	3.4	3.2	3.0

$$f'(1.1) = \frac{3.8 - 4.0}{1.2 - 1.0} = \frac{-0.2}{0.2} = -1 \quad f'(1.7) = \frac{3.4 - 3.6}{1.9 - 1.5} = \frac{-0.2}{0.4} = -\frac{1}{2}$$

Since the derivatives are increasing, the function is concave up.

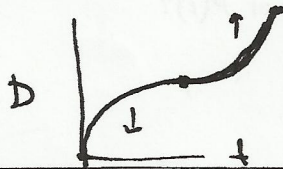
11. (a) Water is flowing at a constant rate into a cylindrical container standing vertically. Sketch a graph showing the depth of water against time.



(b) Water is flowing at a constant rate into a cone-shaped container standing in its vertex. Sketch a graph showing the depth of the water against time.



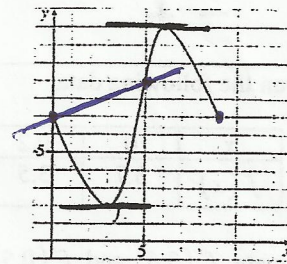
12. If water is flowing at a constant rate into the Grecian urn in the figure on the right, sketch a graph of the depth of the water against time. Mark on the graph the time at which the water reaches the widest point of the urn.



13. Let f be defined as shown in the figure on the right. Use the graph of f to estimate the values of c that satisfy the conclusion of Rolle's Theorem on $[0, 9]$. What theorem would apply for the interval $[0, 5]$?

Yes $c = 3, c = 6$

No Rolle's
MVT OK



14. At what value of x does the graph of $y = \frac{1}{x^2} - \frac{1}{x^3}$ have a point of inflection?

$$f''(x) = \frac{6(x-2)}{x^5}$$

(A) 0

(B) 1

(C) 2

(D) 3

(E) At no value of x

15. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

(A) 4

(B) 2

(C) 1

(D) 0

(E) -2

16. Let f be a polynomial function with degree greater than 2. If $a \neq b$ and $f(a) = f(b) = 1$, which of the following must be true for a least one value of x between a and b ?

I. $f(x) = 0$

II. $f'(x) = 0$

III. $f''(x) = 0$

(A) None

(B) I only

(C) II only

(D) I and II only

(E) I, II, and III

