$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0.$$

Proof:

First, let us manipulate the equation.

$$\frac{1-\cos x}{x} \cdot \frac{1+\cos x}{1+\cos x} = \frac{1-\cos^2 x}{x(1+\cos x)} = \frac{\sin^2 x}{x(1+\cos x)}.$$

Therefore,

$$\lim_{x\to 0} \frac{1-\cos x}{x} = \lim_{x\to 0} \frac{\sin^2 x}{x(1+\cos x)} = \lim_{x\to 0} \left(\frac{\sin x}{x}\right)\left(\frac{\sin x}{1+\cos x}\right) = (1)\left(\frac{0}{2}\right) = 0.$$