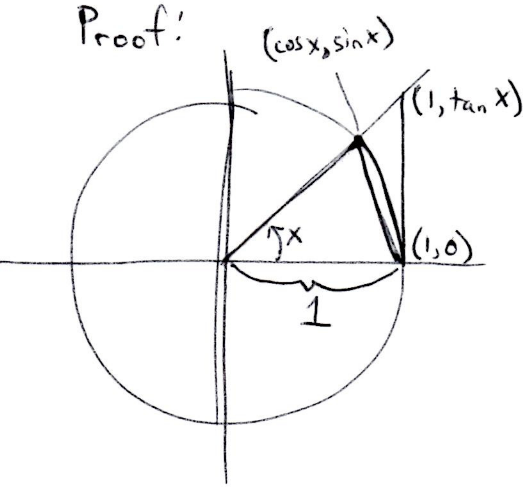
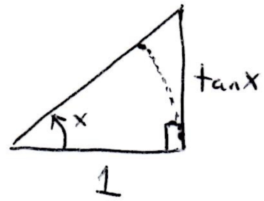
$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

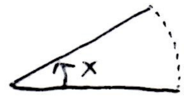
Proof!



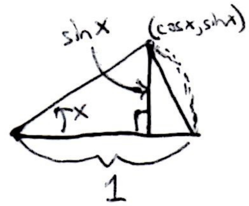
Defining areas: ---- represents the arc of the circle



Area $= \frac{\tan x}{2}$



Area $= \frac{x}{2}$



Area $= \frac{\sin x}{2}$

From these three triangles we have:

(1) $\frac{\tan x}{2} \geq \frac{x}{2} \geq \frac{\sin x}{2}$ multiply everything by 2

(2) $\tan x \geq x \geq \sin x$ divide everything by $\sin x$

(3) $\frac{1}{\cos x} \geq \frac{x}{\sin x} \geq 1$ take the reciprocals

(4) $\cos x \leq \frac{\sin x}{x} \leq 1.$

The limits of each:

$\lim_{x \to 0} \cos x = 1$ $\qquad \lim_{x \to 0} 1 = 1$

Thus, by the Squeeze Theorem, since $1 \leq \lim_{x \to 0} \frac{\sin x}{x} \leq 1$,

$$\lim_{x \to 0} \frac{\sin x}{x} = 1.$$