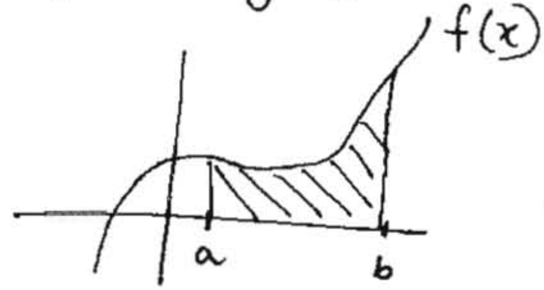
Calculus
Notes 4.3

1-14-13

- Definite Integral as Area of a Region

$$\text{Area} = \int_a^b f(x)\,dx$$



Example

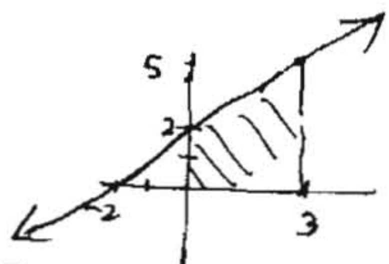
(a) $\int_1^3 4\,dx = \boxed{8}$

$\int(4, x, 1, 3) = 8$

Rectangle

$A = 2(4) = 8$

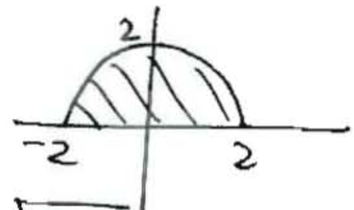
- (b) $\int_0^3 (x+2)\,dx = \frac{21}{2}$



trapezoid

$A = \frac{1}{2}(3)(2+5) = \boxed{\frac{21}{2}}$

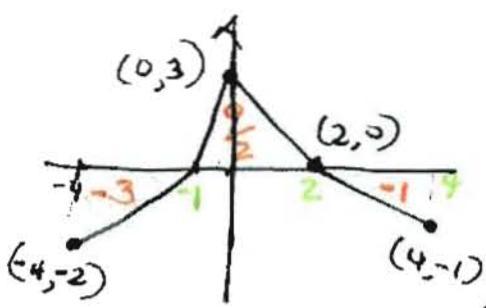
(c) $\int_{-2}^{2} \sqrt{4-x^2}\,dx = 2\pi$



Semi circle

$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(2)^2 = \boxed{2\pi}$

- (d)



$\int_{-4}^{4} f(x)\,dx = \boxed{\frac{1}{2}}$

$A = \frac{1}{2}(b)(h)$

3 triangles

$A = -\left[\frac{1}{2}(3)(2)\right] + \frac{1}{2}(3)(3) - \left[\frac{1}{2}(2)(1)\right]$

$= -3 + \frac{9}{2} - 1 = \boxed{\frac{1}{2}}$

Subtract if below x axis

Definition of 2 Special Definite Integrals

① $\int_a^a f(x)\,dx = 0$ ② $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

Examples

ⓐ $\int_{\pi}^{\pi} \sin x\,dx = \boxed{0}$

ⓑ $\int_3^0 (x+2)\,dx = -\int_0^3 (x+2)\,dx = \boxed{-\frac{21}{2}}$

previous Example $\int_0^3 (x+2)\,dx = \frac{21}{2}$

Properties of Integrals

① $\int_a^a f(x)\,dx = 0$

② $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

③ $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

④ $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$

⑤ $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

Examples

$\frac{1}{2}(1)(1)=\frac{1}{2}$

ⓐ $\int_{-1}^{1} |x|\,dx = \int_{-1}^{0} -x\,dx + \int_{0}^{1} x\,dx$

$= \frac{1}{2} + \frac{1}{2} = \boxed{1}$

ⓑ $\int_{1}^{3} (-x^2+4x-3)\,dx$

Given $\int_{1}^{3} x^2\,dx = \frac{26}{3}$

$\int_{1}^{3} x\,dx = 4$

$\int_{1}^{3} dx = 2$

$\int_{1}^{3} (-x^2+4x-3)\,dx = \int_{1}^{3} (-x^2)\,dx + \int_{1}^{3} 4x\,dx + \int_{1}^{3} (-3)\,dx$

$= -\int_{1}^{3} x^2\,dx + 4\int_{1}^{3} x\,dx - 3\int_{1}^{3} dx$

$= -\frac{26}{3} + 4(4) - 3(2) = \boxed{\frac{4}{3}}$

Given $\int_{0}^{3} f(x)\,dx = 4$ and $\int_{3}^{7} f(x)\,dx = -1$ Find:

ⓐ $\int_{0}^{7} f(x)\,dx$

$= \int_{0}^{3} f(x)\,dx + \int_{3}^{7} f(x)\,dx$

$= 4-1 = \boxed{3}$

ⓑ $\int_{3}^{7} 2f(x)\,dx = 2\int_{3}^{7} f(x)\,dx$

$= 2(-1) = \boxed{-2}$

ⓒ $\int_{5}^{5} f(x)\,dx = 0$

Continued

Given $\int_0^3 f(x)\,dx = 4$ and $\int_3^7 f(x)\,dx = -1$

ⓓ $\int_0^4 f(x+3)\,dx$

Shifted 3 left so

$\int_3^7 f(x)\,dx = -1$

is shifted 3 left to $\int_0^4 f(x+3)\,dx = \boxed{-1}$

ⓔ $\int_{-3}^7 f(x)\,dx$

even $\int_{-3}^0 f(x)\,dx + \int_0^3 f(x)\,dx + \int_3^7 f(x)\,dx = 4+4-1 = \boxed{7}$

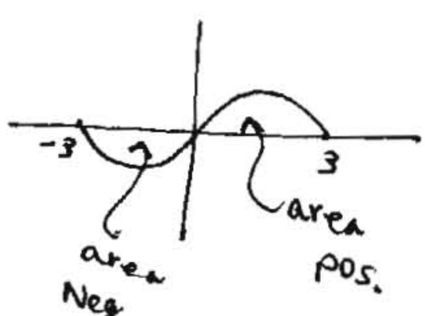
even fcn

area same for even fcn

areas are equal because symmetric

odd $\int_{-3}^0 f(x)\,dx + \int_0^3 f(x)\,dx + \int_2^7 f(x)\,dx = -4+4-1 = \boxed{-1}$

odd fcn



area opp signs