



Related Rates WS

1) Given:  $h = 16 \text{ cm}$ $r = 4 \text{ cm}$ $\frac{dV}{dt} = 2 \frac{\text{cm}^3}{\text{sec}}$  $\frac{h}{r} = \frac{16}{4}$ $h = 4r$ $r = \frac{h}{4}$

a) when $h = 5 \text{ cm}$, $\frac{dh}{dt} = ?$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{h}{4}\right)^2 h = \frac{\pi}{48} h^3 \rightarrow 2 = \frac{\pi}{16} (5)^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{\pi}{16} h^2 \frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = \frac{32}{25\pi} \frac{\text{cm}}{\text{sec}}}$$

b) when $r = 3 \text{ cm}$, $\frac{dr}{dt} = ?$

$$V = \frac{1}{3} \pi r^2 (4r) = \frac{4\pi}{3} r^3$$

$$2 = 4\pi (3)^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\boxed{\frac{dr}{dt} = \frac{1}{18\pi} \frac{\text{cm}}{\text{sec}}}$$

2) $V = \frac{4}{3} \pi r^3$ $\frac{dV}{dt} = 10 \frac{\text{in}^3}{\text{min}}$

when $V = 36\pi \text{ in}^3$: $36\pi = \frac{4}{3} \pi r^3$

$$36 = \frac{4}{3} r^3$$

$$27 = r^3$$

$$r = 3 \text{ in}$$

a) $\frac{dr}{dt} = ?$ $V = \frac{4}{3} \pi r^3$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$10 = 4\pi (3)^2 \frac{dr}{dt}$$

$$\boxed{\frac{dr}{dt} = \frac{5}{18\pi} \frac{\text{in}}{\text{min}}}$$

b) $\frac{dA}{dt} = ?$ $A_s = 4\pi r^2$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 8\pi (3) \left(\frac{5}{18\pi}\right)$$

$$\boxed{\frac{dA}{dt} = \frac{20}{3} \frac{\text{in}^2}{\text{min}}}$$

3) $V = \pi r^2 h + \frac{4}{3} \pi r^3$ $\frac{dV}{dt} = 261\pi \frac{\text{cm}^3}{\text{min}}$ when $r = 3 \text{ cm}$, $V = 144\pi \text{ cm}^3$, $\frac{dr}{dt} = 2 \frac{\text{cm}}{\text{min}}$

$$144\pi = \pi (3)^2 (h) + \frac{4}{3} \pi (3)^3$$

Given: $144 = 9h + 360$

$$\boxed{h = 12 \text{ cm}}$$

a) $\frac{dh}{dt} = ?$ $\frac{dV}{dt} = (2\pi r \frac{dr}{dt})(h) + (\pi r^2) \left(\frac{dh}{dt}\right) + 4\pi r^2 \frac{dr}{dt}$

$$9 \frac{dh}{dt} = 45$$

$$261\pi = 2\pi(3)(2) \frac{dh}{dt} + \pi(3)^2 \left(\frac{dh}{dt}\right) + 4\pi(3)^2(2)$$

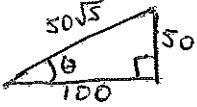
$$261 = 144 + 9 \frac{dh}{dt} + 72$$

$$\boxed{\frac{dh}{dt} = 5 \frac{\text{cm}}{\text{min}}}$$

4) Given: $\frac{dy}{dt} = 3 \frac{m}{sec}$

a) when $y = 50m$, $\frac{dx}{dt} = ?$ $100^2 + y^2 = x^2$
 $100^2 + 50^2 = x^2$ $\sqrt{100^2 + 50^2} = x = 50\sqrt{5} m$
 $2y \frac{dy}{dt} = 2x \frac{dx}{dt}$ $\frac{dx}{dt} = \frac{50}{50\sqrt{5}} (3) = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5} m}{5 sec}$
 $\frac{dx}{dt} = \frac{y}{x} \frac{dy}{dt}$

b) when $y = 50m$, $\frac{dA}{dt} = ?$ $A = \frac{100y}{2} = 50y$
 $x = 50\sqrt{5}$ $\frac{dA}{dt} = 50 \frac{dy}{dt} = 50(3) = 150 \frac{m^2}{s^2}$

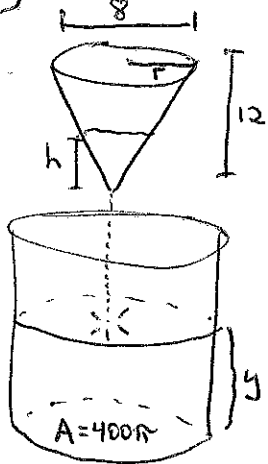
c) when $y = 50m$, $\frac{d\theta}{dt} = ?$ $x = 50\sqrt{5}$

 $\sec \theta = \frac{50\sqrt{5}}{100} = \frac{\sqrt{5}}{2}$ $\sec^2 \theta = \frac{5}{4}$
 $\tan \theta = \frac{y}{x} = \frac{1}{2}$
 $\sec^2 \theta \frac{d\theta}{dt} = \frac{1}{100} \frac{dy}{dt}$ $\left(\frac{5}{4}\right) \frac{d\theta}{dt} = \frac{3}{100}$ $\frac{d\theta}{dt} = \frac{3}{100} \cdot \frac{4}{5}$
 $\frac{d\theta}{dt} = \frac{3}{125} \frac{rad}{sec}$

5) Given: $\frac{dC}{dt} = 6 \frac{in}{sec}$ $P = 8r$ $C = 2\pi r$ $\therefore P = 8\left(\frac{C}{2\pi}\right) = \frac{4C}{\pi}$ or $P = \frac{4}{\pi} C$
 $r = \frac{C}{2\pi}$ $P = 8\left(\frac{C}{2\pi}\right) = \frac{4C}{\pi}$

a) $\frac{dP}{dt} = ?$ $P = \frac{4}{\pi} C$
 $\frac{dP}{dt} = \frac{4}{\pi} \frac{dC}{dt}$ $\frac{4}{\pi} (6) = \frac{24}{\pi} \frac{in}{sec}$

b) when $A_c = 25\pi in^2$, $\frac{dA_{between}}{dt} = ?$ $A = 25\pi = \pi r^2$ $C = 2\pi r$
 $r^2 = 25$ $\frac{dC}{dt} = 2\pi \frac{dr}{dt}$
 $r = 5 in$ $6 = 2\pi \frac{dr}{dt}$
 $\frac{dr}{dt} = \frac{3}{\pi} \frac{in}{sec}$
 $A = (2r)^2 - \pi r^2$
 $A = 4r^2 - \pi r^2$
 $A = (4 - \pi)r^2$
 $\frac{dA}{dt} = 2(4 - \pi)r \frac{dr}{dt}$ $\frac{dA}{dt} = 2(4 - \pi)(5)\left(\frac{3}{\pi}\right)$
 $\frac{dA}{dt} = \left(\frac{120}{\pi} - 30\right) \frac{in^2}{sec}$

b) Given:

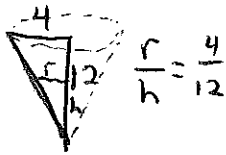


$$\frac{dh}{dt} = (h-12) \frac{ft}{min}$$

a) $V = \frac{1}{3} \pi r^2 h$

$$V = \frac{1}{3} \pi \left(\frac{h}{3}\right)^2 h$$

$$V = \frac{\pi}{27} h^3$$



$$\frac{r}{h} = \frac{4}{12}$$

$$r = \frac{h}{3} \text{ or } h = 3r$$

b) $\frac{dV}{dt} = ?$ when $h=3$

$$V = \frac{\pi}{27} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{9} h^2 (h-12)$$

$$\frac{dV}{dt} = \frac{\pi}{9} (3)^2 (3-12)$$

$$= -9\pi \frac{ft^3}{min}$$

c) ~~$V = \pi r^2 h$~~

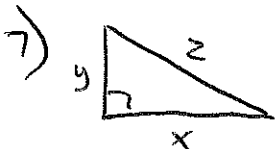
$$V = \pi r^2 y$$

$$V = 400\pi y$$

$$\frac{dV}{dt} = 400\pi \frac{dy}{dt}$$

$$9\pi = 400\pi \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{9}{400} \frac{ft}{min}$$



when $x=4$
 $y=3$

then $z=5$

$$\frac{dz}{dt} = 1 \quad \frac{dx}{dt} = 3 \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{1}{3} \frac{dx}{dt}$$

$$x^2 + y^2 = z^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

B

$$4 \frac{dx}{dt} + 3 \left(\frac{1}{3} \frac{dx}{dt}\right) = 5(1)$$

$$5 \frac{dx}{dt} = 5$$

$$\frac{dx}{dt} = 1$$

8) Given: $\frac{dV}{dt} = 16 \frac{\text{ft}^3}{\text{min}}$, $\frac{dh}{dt} = 4 \frac{dr}{dt}$, $h=8$, $r=2$

$$V = \frac{1}{3} \pi r^2 h$$

$$\frac{dV}{dt} = \left(\frac{1}{3} \pi r^2\right) \frac{dh}{dt} + \left(\frac{2}{3} \pi r\right) \frac{dr}{dt} h$$

$$16 = \frac{1}{3} \pi (2)^2 \left(4 \frac{dr}{dt}\right) + \frac{2}{3} \pi (2) (8) \frac{dr}{dt}$$

$$16 = \frac{16\pi}{3} \frac{dr}{dt} + \frac{32\pi}{3} \frac{dr}{dt}$$

$$16 = 16\pi \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{\pi} \frac{\text{ft}}{\text{min}}$$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi (2) \left(\frac{1}{\pi}\right)$$

$$\frac{dA}{dt} = 4 \frac{\text{ft}^2}{\text{min}}$$

