

9/30 - SOE 3 variables

$$\begin{aligned}
 1) \quad & 4x + 5y + 3z = 15 \\
 & x - 3y + 2z = -6 \\
 & \begin{cases} -x + 2y - z = 3 \\ \rightarrow x = 2y - z - 3 \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 & 4(2y - z - 3) + 5y + 3z = 15 \\
 & 8y - 4z - 12 + 5y + 3z = 15 \\
 & \underline{13y - z = 27}
 \end{aligned}$$

$$\begin{aligned}
 & (2y - z - 3) - 3y + 2z = -6 \\
 & \underline{-y + z = -3}
 \end{aligned}$$

$$13y - z = 27$$

$$\underline{-y + z = -3}$$

$$12y = 24$$

$$y = 2$$

$$-(2) + z = -3$$

$$z = -1$$

$$x = 2(2) - (-1) - 3$$

$$x = 4 + 1 - 3$$

$$x = 2$$

$$\boxed{(2, 2, -1)}$$

$$\begin{aligned}
 2) \quad & x + 2y = -1 \rightarrow x = -2y - 1 \\
 & 3x - y + 4z = 17 \\
 & -4x + 2y - 3z = -30
 \end{aligned}$$

$$3(-2y - 1) - y + 4z = 17$$

$$-6y - 3 - y + 4z = 17$$

$$\underline{-7y + 4z = 20}$$

$$-4(-2y - 1) + 2y - 3z = -30$$

$$8y + 4 + 2y - 3z = -30$$

$$\underline{10y - 3z = -34}$$

$$3(-7y + 4z = 20) \quad -21y + 12z = 60$$

$$4(10y - 3z = -34) + 40y - 12z = -136$$

$$19y = -76$$

$$y = -4$$

$$-7(-4) + 4z = 20$$

$$28 + 4z = 20$$

$$4z = -8$$

$$z = -2$$

$$x = -2(-4) - 1$$

$$x = 8 - 1$$

$$x = 7$$

$$\boxed{(7, -4, -2)}$$

$$3) \begin{cases} 2x - y + 4z = 19 \\ -x + 3y - 2z = -7 \\ 4x + 2y + 3z = 37 \end{cases} \rightarrow y = 2x + 4z - 19$$

$$-x + 3y - 2z = -7$$

$$-x + 3(2x + 4z - 19) - 2z = -7$$

$$4x + 2(2x + 4z - 19) + 3z = 37$$

$$4x + 2y + 3z = 37$$

$$-x + 6x + 12z - 57 - 2z = -7$$

$$4x + 4x + 8z - 38 + 3z = 37$$

$$\underline{5x + 10z = 50}$$

$$\underline{8x + 11z = 75}$$

$$8(5x + 10z = 50)$$

$$5(8x + 11z = 75) \Rightarrow \begin{cases} 40x + 80z = 400 \\ -(40x + 55z = 375) \end{cases}$$

$$5x + 10(1) = 50$$

$$y = 2(8) + 4(1) - 19$$

$$5x = 40$$

$$y = 16 + 4 - 19$$

$$x = 8$$

$$y = 1$$

$$\boxed{(8, 1, 1)}$$

$$25z = 25$$

$$z = 1$$

$$4) \left(\frac{1}{3}x + \frac{5}{6}y + \frac{2}{3}z = \frac{4}{3} \right) 6$$

$$2x + 5y + 4z = 8$$

$$2x + 5(-4x - 9z + 8) + 4z = 8$$

$$\left(\frac{1}{6}x + \frac{2}{3}y + \frac{1}{4}z = \frac{5}{6} \right) 12$$

$$2x + 8y + 3z = 10$$

$$2x - 20x - 45z + 40 + 4z = 8$$

$$\underline{-18x - 41z = -32}$$

$$\left(\frac{2}{3}x + \frac{1}{6}y + \frac{3}{2}z = \frac{4}{3} \right) 6$$

$$\begin{cases} 4x + y + 9z = 8 \\ y = -4x - 9z + 8 \end{cases}$$

$$2x + 8(-4x - 9z + 8) + 3z = 10$$

$$2x - 32x - 72z + 64 + 3z = 10$$

$$\underline{-30x - 69z = -54}$$

$$30(18x + 41z = 32)$$

$$540x + 1230z = 960$$

$$18(30x + 69z = 54)$$

$$\Rightarrow \begin{cases} 540x + 1230z = 960 \\ -(540x + 1242z = 972) \end{cases}$$

$$-12z = -12$$

$$y = -4\left(\frac{-1}{2}\right) - 9(1) + 8$$

$$y = 2 - 9 + 8$$

$$18x + 41(1) = 32$$

$$z = 1$$

$$y = 1$$

$$18x = -9$$

$$x = -\frac{1}{2}$$

$$\boxed{\left(-\frac{1}{2}, 1, 1\right)}$$

$$5) \begin{cases} (-1) + 2(2) - 3(-3) = 12 \\ -(-1) - (-2) + (-3) = -4 \\ 2(-1) + 3(2) - 2(-3) = 10 \end{cases}$$

$$\boxed{a=12 \quad b=-4 \quad c=10}$$