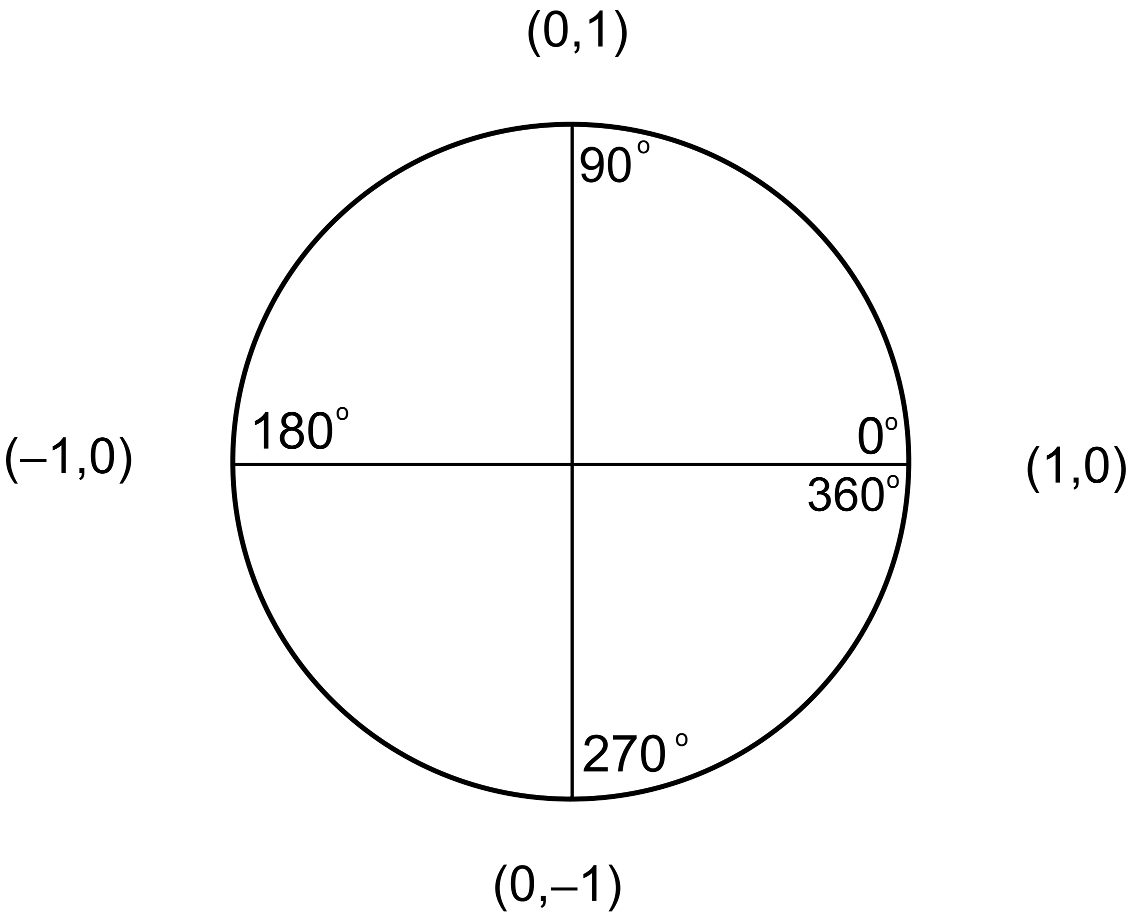
The Unit Circle

To begin . . .

fold a flattened paper plate into quarters

To be a ***one unit circle*** means the radius is 1 unit in every direction. Label the coordinates as shown and include the degrees.



Convert to radians:

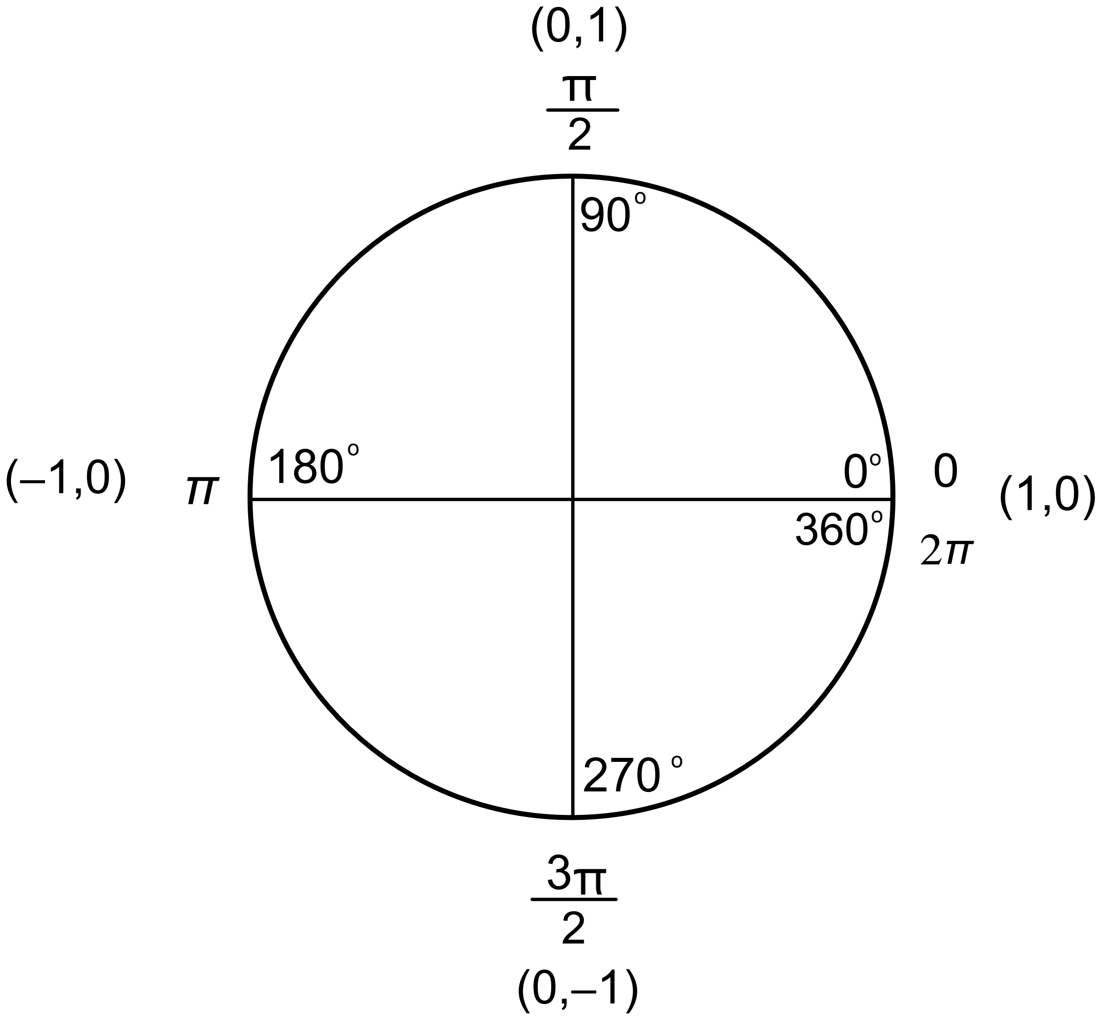
$90^{\circ}\left(\frac{\pi}{180^{\circ}}\right) = \frac{\pi}{2}$

$180^{\circ}\left(\frac{\pi}{180^{\circ}}\right) = \pi$

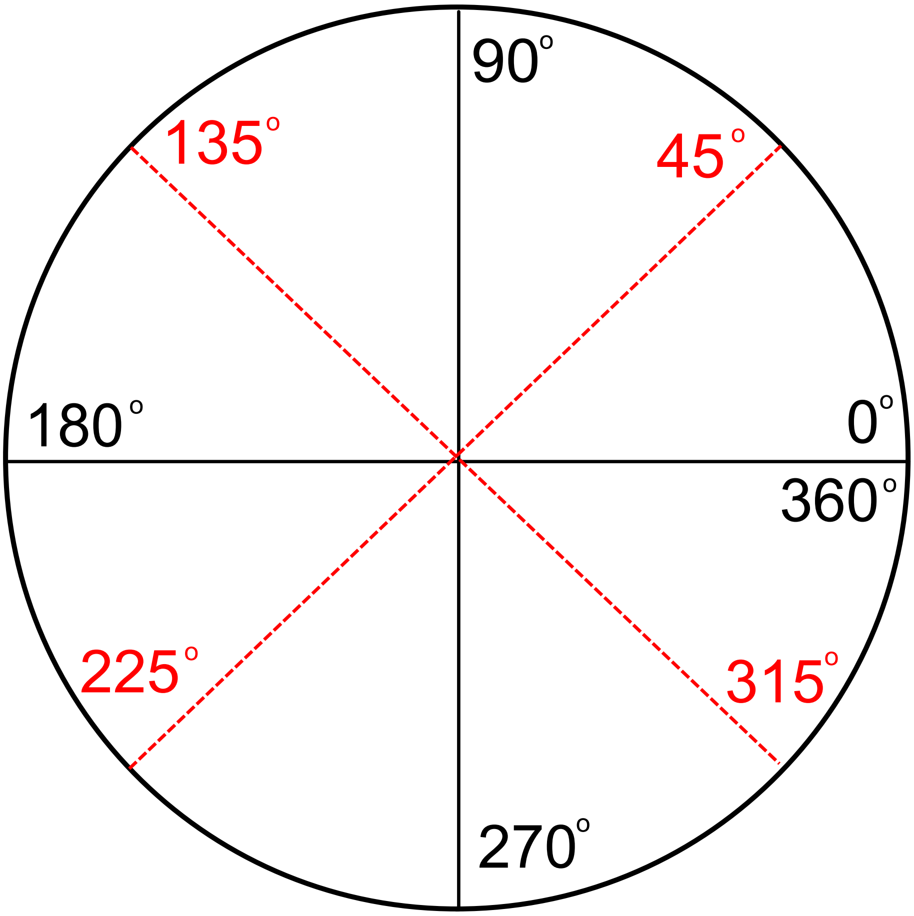
$270^{\circ}\left(\frac{\pi}{180^{\circ}}\right) = \frac{3\pi}{2}$

$360^{\circ}\left(\frac{\pi}{180^{\circ}}\right) = 2\pi$

Then place the radian measurements next to their corresponding degrees.


(0,1)
π/2
90°
(–1,0)
π
180°
0°
0
(1,0)
360°
2π
270°
3π/2
(0,–1)

Fold the paper plate into eighths to create 45° wedges.



Convert to radians:

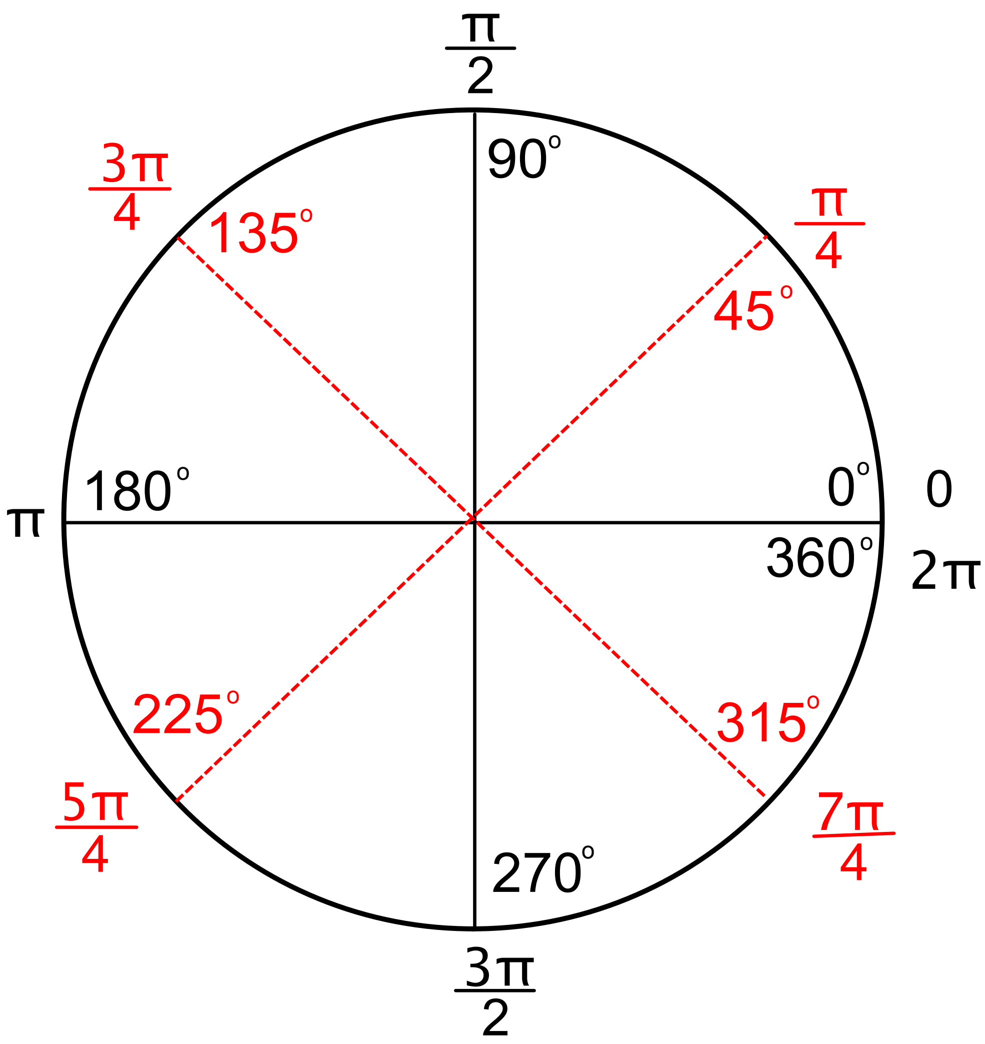
$45^{\circ}\left(\frac{\pi}{180^{\circ}}\right) = \frac{\pi}{4}$

$135^{\circ}\left(\frac{\pi}{180^{\circ}}\right) = \frac{3\pi}{4}$

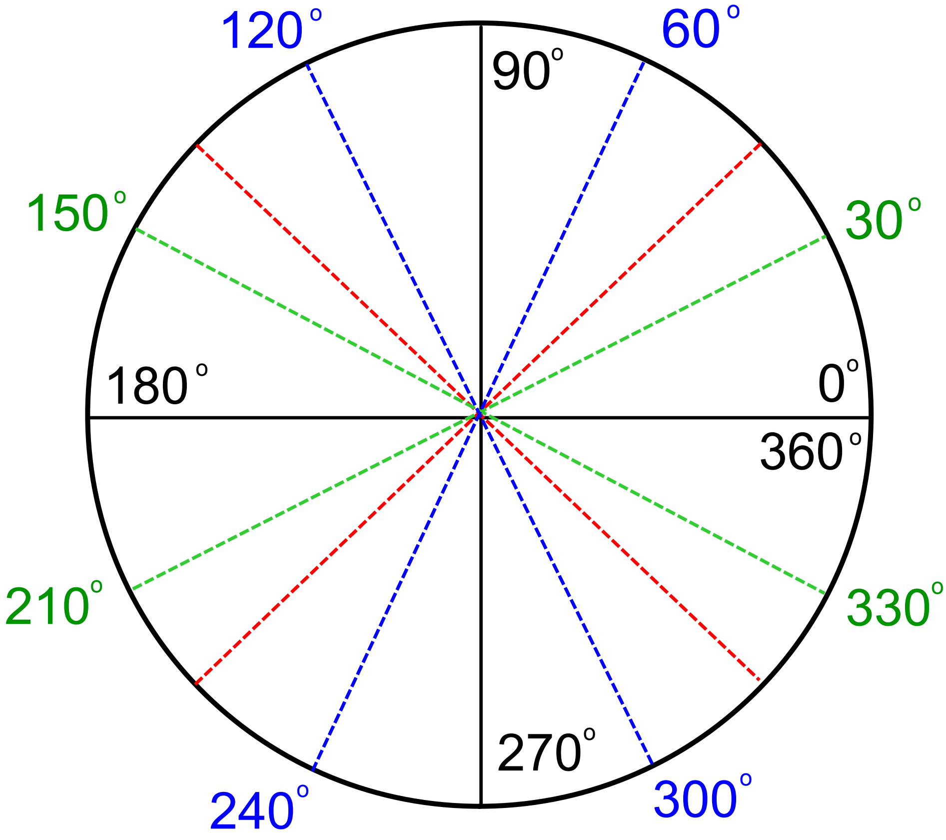
$225^{\circ}\left(\frac{\pi}{180^{\circ}}\right) = \frac{5\pi}{4}$

$315^{\circ}\left(\frac{\pi}{180^{\circ}}\right) = \frac{7\pi}{4}$

Then place the radian measurements next to their corresponding degrees.


π/2
90°
3π/4
135°
π/4
45°
π
180°
0°
0
360°
2π
225°
315°
5π/4
7π/4
270°
3π/2

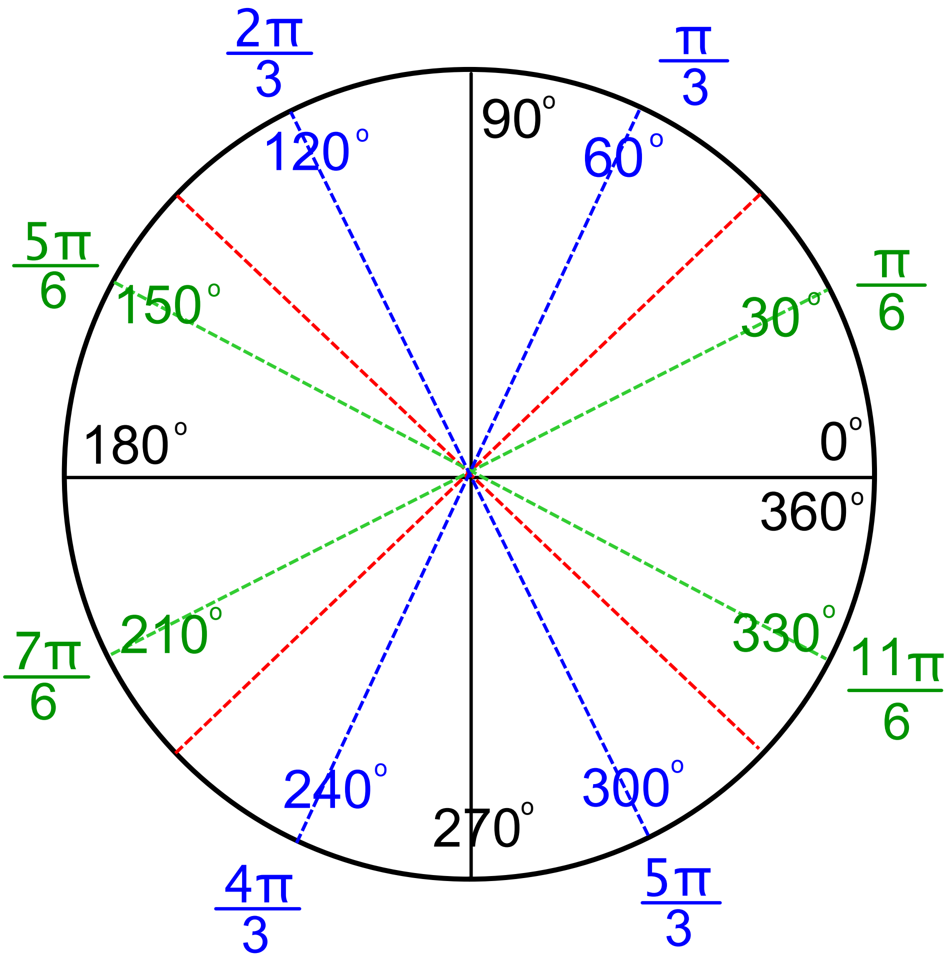
Next we need folds for 30° and 60°. These are a little more challenging.



Next, convert each angle to find its corresponding radian value.

$30^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{6}$	$210^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{7\pi}{6}$
$60^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{3}$	$240^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{4\pi}{3}$
$120^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{2\pi}{3}$	$300^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{5\pi}{3}$
$150^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{5\pi}{6}$	$330^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{11\pi}{6}$

Then place the radian measurements next to their corresponding degrees.

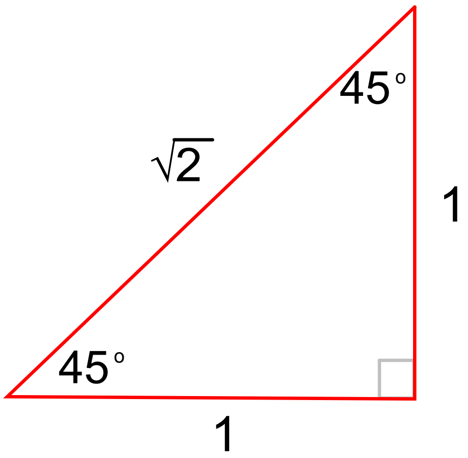

90°
180°
0°
360°
270°
60°
π/3
120°
2π/3
240°
4π/3
300°
5π/3
30°
π/6
150°
5π/6
210°
7π/6
330°
11π/6

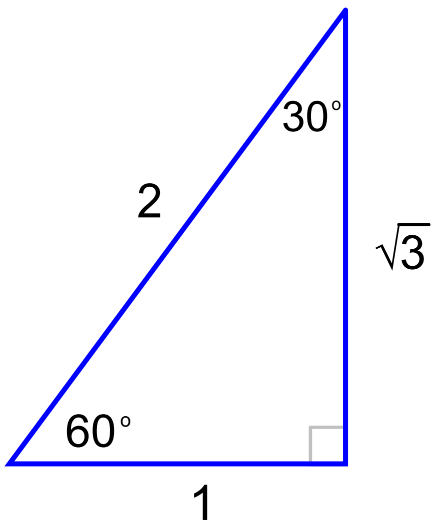
Next, let's find the actual points on the circumference of the unit circle as (*x*, *y*) points for each angle.

It's really quite simple there are only five lengths to know....

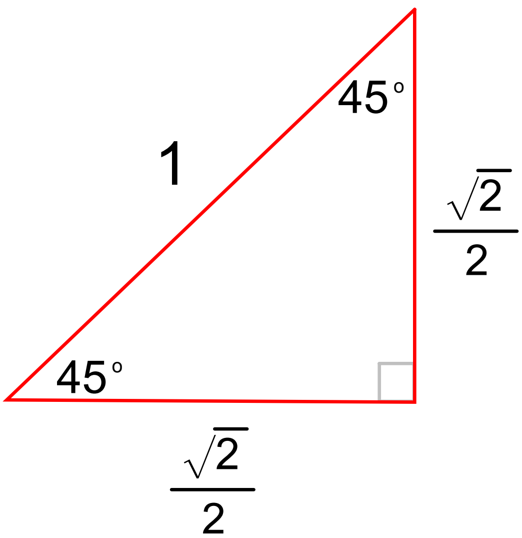
$$0 \quad \frac{1}{2} \quad \frac{\sqrt{2}}{2} \quad \frac{\sqrt{3}}{2} \quad 1$$

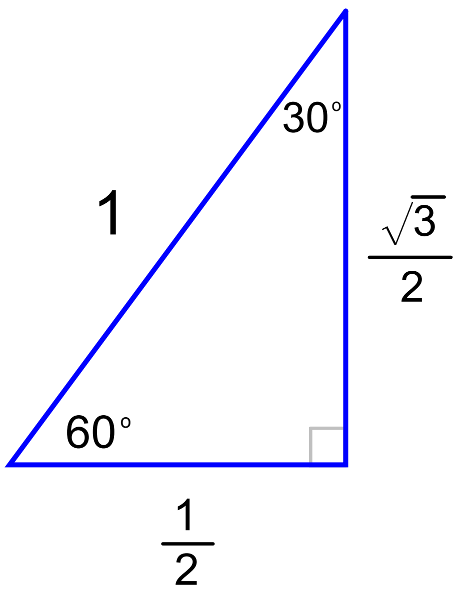
Remember the special triangles from Geometry ???



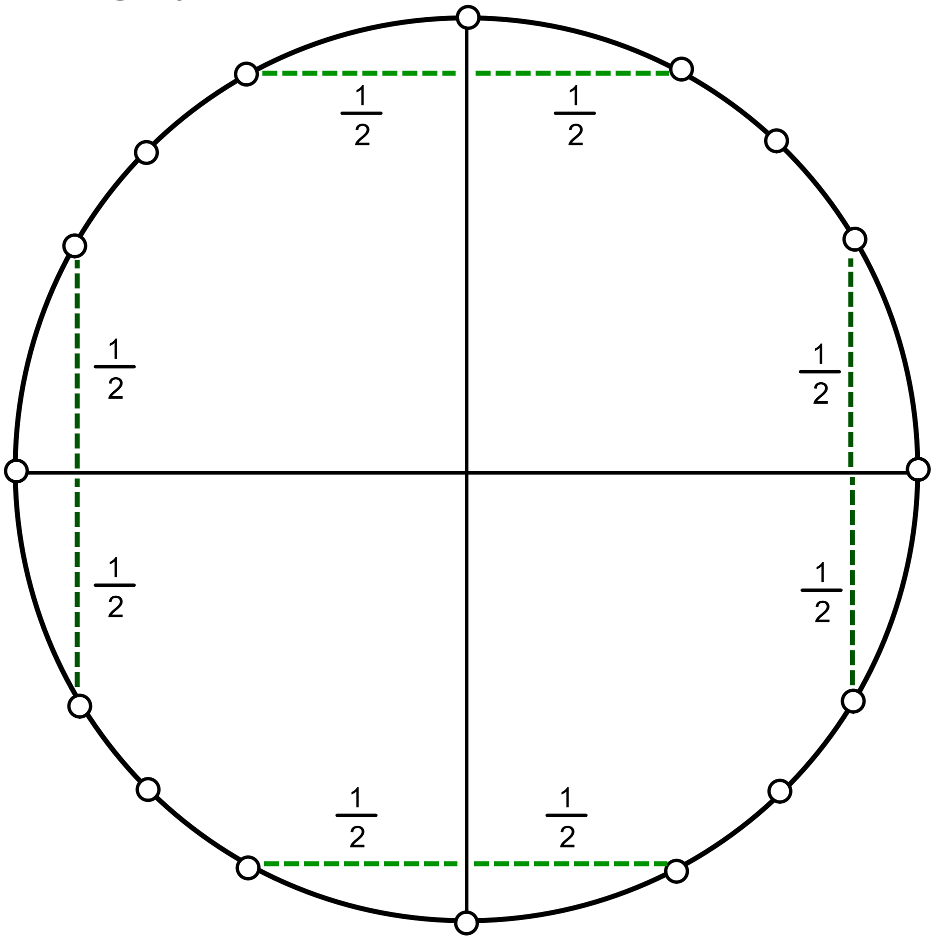


Let's force the hypotenuse to be 1 unit in length... Can you do it?

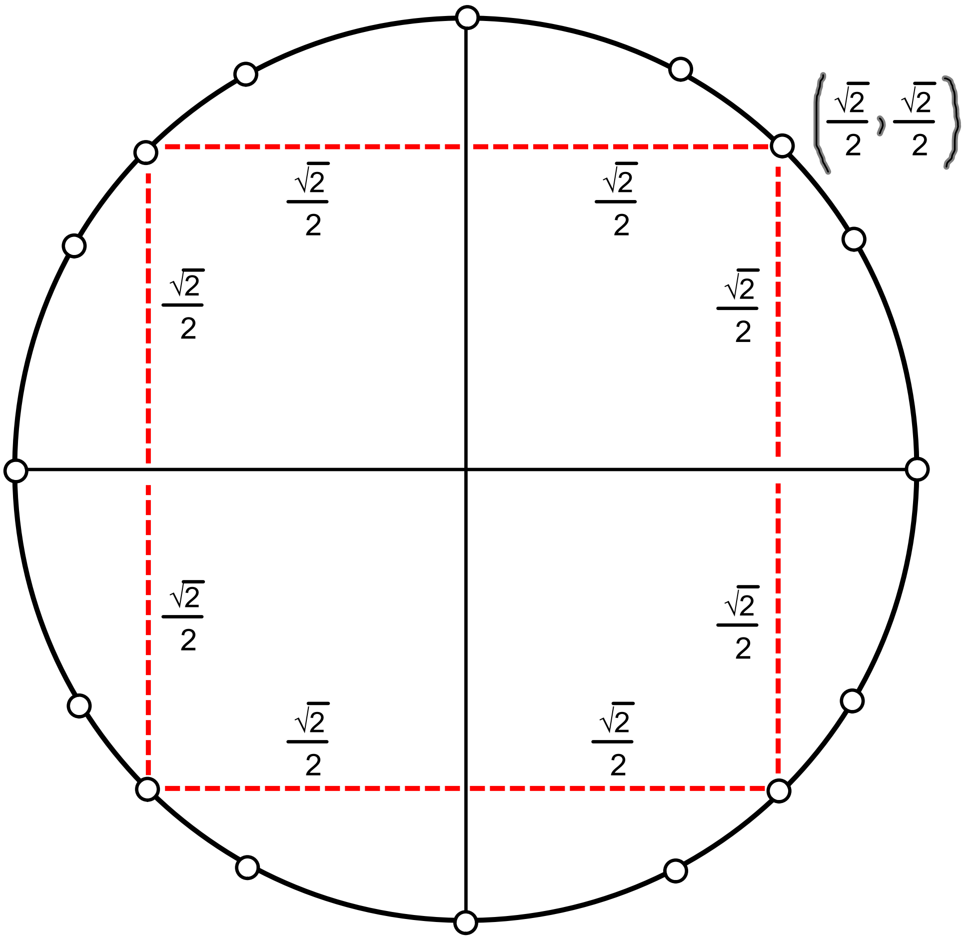




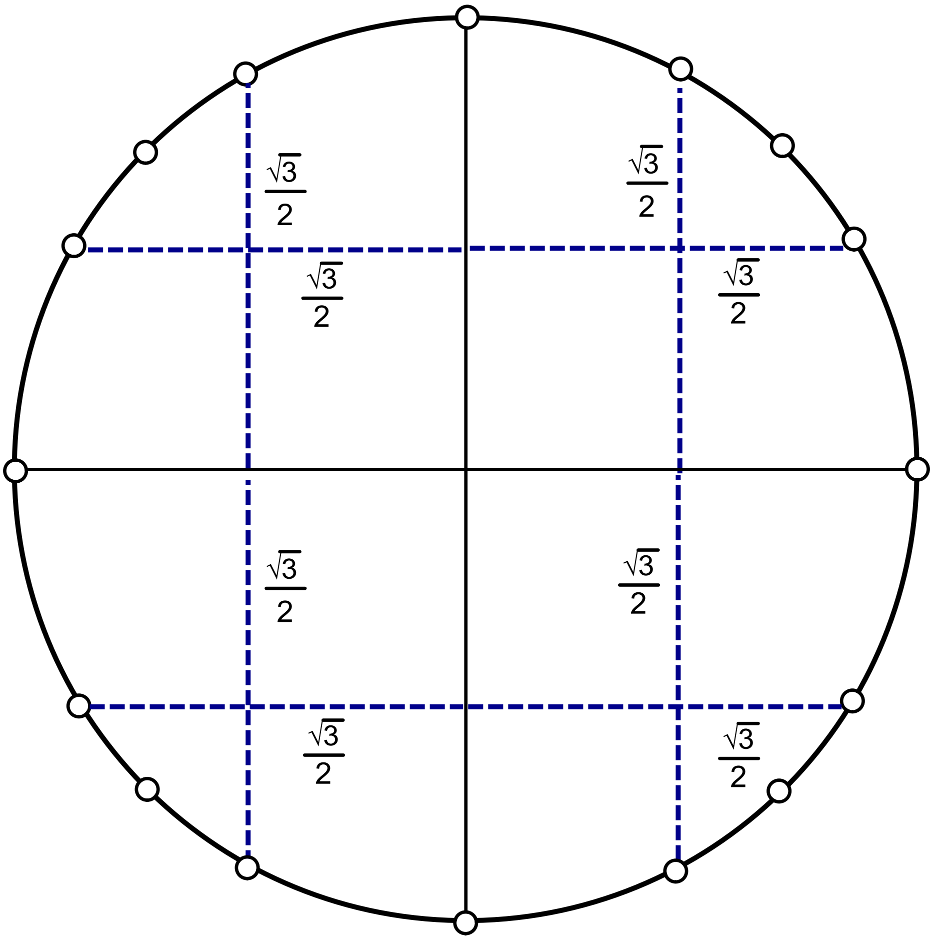
The short guys . . .



The medium guys . . .

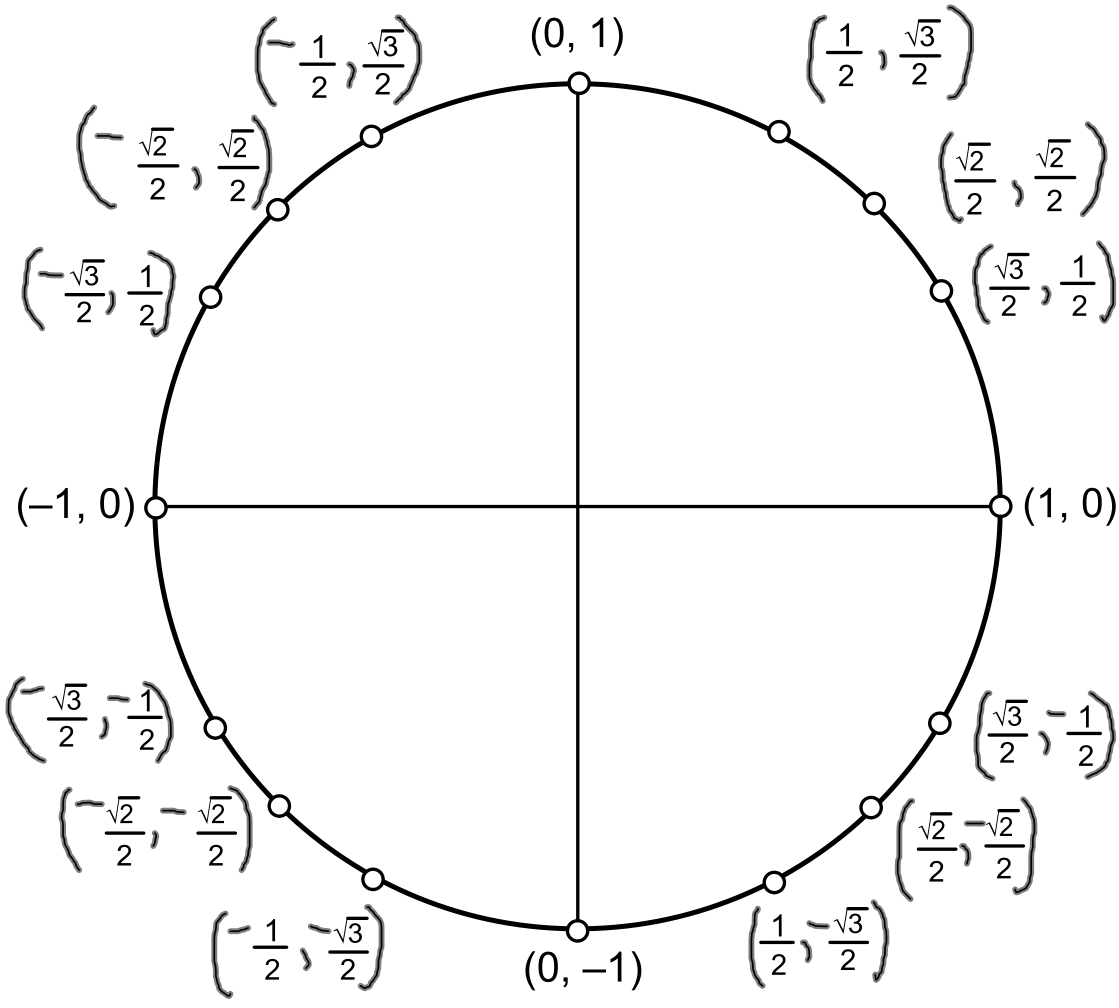


The tall guys . . .



The final steps . . .

First, let's place the guys by their size,
then we can figure out the signs. . .



Now. . . The best part....

- The ***x-coordinates*** are the exact values of the ***cosine*** for each angle on the unit circle.
- The **reciprocal of each *x-coordinate*** is the exact value of the ***secant*** for each angle on the unit circle.
- The ***y-coordinates*** are the exact values of the ***sine*** for each angle on the unit circle.
- The **reciprocal of each *y-coordinate*** is the exact value of the ***cosecant*** for each angle on the unit circle.
- The ratio of ***y/x*** is the exact value of the ***tangent*** for each angle on the unit circle.
- The ratio of ***x/y*** is the exact value of the ***cotangent*** for each angle on the unit circle.