1) X = "Old" hours to complete job X speed = $\frac{1 \text{ job}}{21 \text{ hours}}$ or $\frac{1}{21}$ job per hour

Y = "New" hours to complete job Y speed = $\frac{1 \text{ job}}{15 \text{ hours}}$ or $\frac{1}{15}$ job per hour

Add speeds for speed of both for 1 job. $\frac{1}{21} + \frac{1}{15} = \frac{4}{35} \frac{\text{jobs}}{\text{hour}}$ convert $\frac{\frac{35}{4} \text{ hours}}{\text{job}}$ = **8.75 hours**

2) (Joe)(4.5) = (Bill)

J = Joe's hours

B = Bill's hours

$4.5J = B$ or $J = \frac{B}{4.5}$

$\frac{1}{J} + \frac{1}{B} = \frac{1}{10}$

$\left(\frac{1}{J} + \frac{1}{4.5J} = \frac{1}{10}\right) 45J$

$45 + 10 = 4.5J$

$55 = 4.5J$

$J = \frac{110}{9}$ or $12\frac{2}{9}$ hours

$\frac{1}{B/4.5} + \frac{1}{B} = \frac{1}{10}$

$10B\left(\frac{4.5}{B} + \frac{1}{B} = \frac{1}{10}\right)$

$45 + 10 = B$

$55 = B$

B = 55 hours

Check work!

$\frac{1}{110/4} + \frac{1}{55} \overset{\checkmark}{=} \frac{1}{10}$

3) H = hot time (minutes)

C = cold time (minutes)

D = drain time (minutes)

T = total time (minutes)

$\frac{1}{H} + \frac{1}{C} - \frac{1}{D} = \frac{1}{T}$

$\frac{1}{8} + \frac{1}{6} - \frac{1}{4} = \frac{1}{T}$

$\frac{1}{24} = \frac{1}{T}$

T = 24 minutes

4) X = time up hill

y = time down hill

$x + y = 1\text{hr} + 20\text{min}$

$x + y = \frac{4}{3}\text{hr}$

both distances the same

$6x = 10y$

~~$6x = 10(\frac{4}{3} - x)$~~

$6(\frac{4}{3} - y) = 10y$

$8 - 6y = 10y$

$8 = 16y$

$y = \frac{1}{2}\text{hr}$

$x = \frac{4}{3} - \frac{1}{2} = \frac{5}{6}\text{hr}$

$D = rt$

$D = \underbrace{6\left(\frac{5}{6}\right)}_{\text{up}} + \underbrace{10\left(\frac{1}{2}\right)}_{\text{down}} = 5 + 5 =$ **10 km**

5) X = time at $45\frac{km}{hr}$ $\quad X+y = 2hr+40min$

y = time at $75\frac{km}{hr}$ $\quad X+y = \frac{8}{3}hr$

$\underbrace{45X}_{\text{1st Distance}} + \underbrace{75y}_{\text{2nd distance}} = \underbrace{150}_{\text{total distance}}$

$45X + 75(\frac{8}{3}-X) = 150$

$45X + 200 - 75X = 150$

$-30X = -50$

$X = \frac{5}{3}hr \quad y = 1hr$

1st distance: $45(\frac{5}{3}) = \boxed{75\ km}$

2nd distance: $75(1) = \boxed{75\ km}$

6) X = speed of current $\frac{km}{hr}$

rate upstream $= 15-X$

rate downstream $= 15+X$

$D=rt \therefore t=\frac{D}{r}$ $\quad$ add times for total time $(\frac{24}{5}hr)$

$t_{upstream} = \frac{35}{15-X}$

$t_{downstream} = \frac{35}{15+X}$

$\frac{35}{15-X} + \frac{35}{15+X} = \frac{24}{5}$

$35\left(\frac{1}{15-X} + \frac{1}{15+X}\right) = \frac{24}{5}$

$(15-X)(15+X)\left(\frac{1}{15-X} + \frac{1}{15+X} = \frac{24}{175}\right)$

$(15+X) + (15-X) = \frac{24}{175}(225-X^2)$

$30 = \frac{24}{175}(225-X^2)$

$\frac{875}{4} = 225-X^2$

$X^2 = 225 - \frac{875}{4}$

$X^2 = 6.25$

$\boxed{X = 2.5\ \frac{km}{hr}}$

7) total distance $= 40\ km$ $\quad D=rt \quad t=\frac{D}{r}$ $\quad$ add times for total time

$\frac{10}{30} + \frac{30}{15} = t$

$\frac{1}{3} + 2 = t = 2hr\ 20min$

vs. $d = 40\ km$, $r = 20\ \frac{km}{hr}$ $\quad t = \frac{40}{20} = 2hrs$

difference of $\boxed{20\ min}$

8) J = Julien's time (20 min)

T = Julien's and Remy's time (11 min)

R = Remy's time

$\frac{1}{J} + \frac{1}{R} = \frac{1}{T}$

$\frac{1}{20} + \frac{1}{R} = \frac{1}{11}$

$\frac{1}{R} = \frac{1}{11} - \frac{1}{20} = \frac{9}{220}$

$R = \frac{220}{9} = \boxed{24\frac{4}{9}\ min}$

10) X = Kyle's speed $\left(\frac{km}{h}\right)$ in still water

$r_{up} = X-3$ $\quad t = 5hr\ 20min = \frac{16}{3}hr$

$r_{down} = X+3$

$$\frac{D_{up}}{r_{up}} + \frac{D_{down}}{r_{down}} = t$$

$$\frac{12}{X-3} + \frac{12}{X+3} = \frac{16}{3}$$

$$3(X+3)(X-3)\left(\frac{12}{X-3} + \frac{12}{X+3} = \frac{16}{3}\right)$$

$$(36X+108)+(36X-108) = 16X^2 - 144$$

$$72X = 16X^2 - 144$$

$$16X^2 - 72X - 144 = 0$$

$$2X^2 - 9X - 18 = 0$$

$$(2X+3)(X-6) = 0$$

$$\boxed{X = 6\ \frac{km}{hr}}$$

$D = 6\left(\frac{16}{3}\right) = \boxed{32\ km}$ in still water

11) G = glassblower's time = 2 hr $\quad A$ = apprentice's time $\quad t = G + A = 1.5hr$

$$\frac{1}{G} + \frac{1}{A} = \frac{1}{t}$$

$$\frac{1}{2} + \frac{1}{A} = \frac{1}{1.5} \Rightarrow \frac{1}{2} + \frac{1}{A} = \frac{2}{3}$$

$$6A\left(\frac{1}{2} + \frac{1}{A} = \frac{2}{3}\right) \Rightarrow 3A + 6 = 4A$$

$$\boxed{A = 6\ hr}$$

12) X = speed of jet stream $\left(\frac{km}{h}\right)$

t = noon to 2:24 p.m = 2.4hr $= \frac{12}{5}hr$

$$\frac{800}{750-X} + \frac{800}{750+X} = \frac{12}{5}$$

$$5(750-X)(750+X)\left(\frac{800}{750-X} + \frac{800}{750+X} = \frac{12}{5}\right) \Rightarrow$$

$$4000(750+X) + 4000(750-X) = 12(562500 - X^2)$$

$$6000000 = 6750000 - 12X^2$$

$$12X^2 = 750000$$

$$X^2 = 62500$$

$$\boxed{X = 250\ \frac{km}{h}}$$

13) Walking distance = Bus distance = d miles

$d = rt$ so $t = \frac{d}{r}$ $\quad t = 12.5 = \frac{25}{2}$ hrs

$$\frac{d}{3} + \frac{d}{12} = \frac{25}{2} \qquad 12\left(\frac{d}{3} + \frac{d}{12} = \frac{25}{2}\right) \Rightarrow 4d + d = 150$$

$$5d = 150$$

$$\boxed{d = 30 \text{ miles}}$$

14) Pipe A: $\frac{1}{6}\frac{\text{tank}}{\text{hr}} \cdot 2 \text{ hr} = \frac{1}{3}$ tank

$1 - \frac{1}{3} = \frac{2}{3}$ of a tank left $\qquad$ from noon to $5\frac{1}{3}$ hr

Pipe B: $\frac{1}{8}\frac{\text{tank}}{\text{hr}} \cdot t \text{ hr} = \frac{2}{3}$ tank $\qquad \boxed{5:20 \text{ PM}}$

$$\frac{t}{8} = \frac{2}{3}$$

$$t = \frac{16}{3} \text{ hr} = 5\tfrac{1}{3} \text{ hr}$$