10) $\lim_{x \to 2} \frac{2x^3 - 16}{x-2} = \lim_{x \to 2} \frac{2(x^3-8)}{x-2} = \lim_{x \to 2} \frac{2[x-2)(x^2+2x+4)}{x-2}$

$= 2[(2)^2 + 2(2) + 4) = 2[4+4+4) = \boxed{24}$

11) $\lim_{x \to \frac{\pi}{2}} \frac{\sin x - 1}{x - \frac{\pi}{2}} \frac{(\sin x + 1)}{(\sin x + 1)} = \lim_{x \to \frac{\pi}{2}} \frac{\sin x^2 - 1}{x \sin x + x - \frac{\pi}{2}\sin x - \frac{\pi}{2}}$

$= \frac{(\sin \frac{\pi}{2})^2 - 1}{\frac{\pi}{2}(\sin \frac{\pi}{2} + \frac{\pi}{2} - \frac{\pi}{2}\sin \frac{\pi}{2} - \frac{\pi}{2})} = \frac{0}{\frac{\pi}{2}(1 - \frac{\pi}{2})} = \frac{0}{\frac{\pi}{2} - \frac{\pi}{4}} = \boxed{0}$

WS Implicit

1) $\frac{dy}{dx}\left[x^3 + xy + y^3 = 1\right] = 3x^2 + x\frac{dy}{dx} + y + 3y^2\frac{dy}{dx} = 0$

$\boxed{\frac{dy}{dx} = \frac{-3x^2 - y}{x + 3y^2}}$

2) $y - x\sin y = 3$

$\frac{dy}{dx} - x\cos y \frac{dy}{dx} - \sin y = 0$

$\frac{dy}{dx} = \frac{- -\sin y}{1 - x\cos y}$

3) $x + \tan(xy) = 0$

$1 + \sec^2(xy) \cdot \left[x\frac{dy}{dx} + y\right] = 0$

$\frac{dy}{dx} = \left(\frac{-1}{\sec^2 xy} - y\right)\frac{1}{x}$

$\sec^2 xy \left[x\frac{dy}{dx} + y\right] = -1$

$= \boxed{\frac{-1 - y\sec^2 xy}{x\sec^2 xy}}$

$x\frac{dy}{dx} + y = \frac{-1}{\sec^2 xy}$

4) If $y = xy + x^2 + 1$ $\qquad y = -y + 1 + 1$ $\qquad 2y = 2$ $\qquad \boxed{y = 1}$

$y = -y + 2$

$$\frac{dy}{dx} \Rightarrow \quad \frac{dy}{dx} = x\frac{dy}{dx} + y + 2x \qquad \frac{dy}{dx} - x\frac{dy}{dx} = y + 2x$$

$$\frac{dy}{dx} = \frac{y+2x}{1-x} \qquad \boxed{x = -1} \qquad \frac{dy}{dx} = \frac{1-2}{1+1} = \boxed{\frac{-1}{2}}$$

5) $$\frac{dy}{dx}\left[y^2 + 2y = 2x + 1\right] \Rightarrow \quad 2y\frac{dy}{dx} + 2\frac{dy}{dx} = 2$$

$$\frac{dy}{dx}(2y+2) = 2$$

$$\frac{dy}{dx} = \frac{(y+1)(0) - \left[1\left(\frac{dy}{dx} + 0\right)\right]}{(y+1)^2} \qquad \frac{dy}{dx} = \frac{2}{2y+2} = \boxed{\frac{1}{y+1}}$$

$$y' = \frac{1}{y+1} = y + 1$$

$$y'' = \frac{(y+1) - 1\left(\frac{dy}{dx}\right)}{(y+1)^2} = \frac{y+1-y'}{(y+1)^2} = y''$$

$$\frac{\frac{y+1}{1} - \frac{1}{y+1}}{(y+1)^2} = y'' \qquad \frac{(y+1)^2 - 1}{y+1} \cdot \frac{1}{(y+1)^2} = \frac{(y+1)^2 - 1}{(y+1)^3} = y''$$

$$\frac{((y+1)+1)((y+1)-1)}{(y+1)^3} = \frac{(y+2)(y)}{(y+1)^3} = \frac{y}{(y+1)^2}$$

$$y' = \frac{1}{y+1} \qquad y'' = \frac{(y+1)(0) - (1)\left(\frac{dy}{dx}\right)}{(y+1)^2}$$

$$= -\frac{1}{y+1} \cdot \frac{1}{(y+1)^2} = \boxed{\frac{-1}{(y+1)^3}}$$

b) $x^3 + y^3 = 8$

$3x^2 + 3y^2 y' = 0$

$y' = \dfrac{-3x^2}{3y^2} = \boxed{\dfrac{-x^2}{y^2}}$

$-x^2 y^{-2}$

$-x^2 - 2y^{-3}\frac{dy}{dx} + y^{-2} - 2x$

$\dfrac{2x^2}{y^3}\dfrac{dy}{dx} - \dfrac{2x}{y^2}$

$\dfrac{x^2}{y^3}\left(\dfrac{-x^2}{y^2}\right) - \dfrac{2x}{y^2}$

$\dfrac{-2x^4}{y^5} - \dfrac{2x}{y^2}$

$y'' = -x^2 - 2y^{-3}y' + y^{-2} - 2x$

$= -x^2 - 2y^{-3}\left(\dfrac{-x^2}{y^2}\right) + \dfrac{-2x}{y^2}$

$= \dfrac{-2x^4}{y^5} + \dfrac{-2x}{y^2} = \dfrac{-2x^4y^2 + -2xy^5}{y^5(y^2)}$

$= \dfrac{-2x^4 + -2xy^3}{y^5} = \dfrac{-2x(x^3+y^3)}{y^5}$

$= \dfrac{-2x(8)}{y^5} = \boxed{\dfrac{-16x}{y^5}}$

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7) $y + \cos y = x + 1$

$\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} - \sin y \dfrac{dy}{dx} = 1$

a) $\dfrac{dy}{dx} = \dfrac{1}{1-\sin y}, \; y \neq$

$1 - \sin y \neq 0$

$-\sin y \neq -1$

$\sin y \neq 1$

$y \neq \frac{\pi}{2}$

b)

$y = \frac{\pi}{2}$

$\frac{\pi}{2} + \cos\frac{\pi}{2} = x + 1$

$\frac{\pi}{2} + 0 = x + 1$

$\boxed{\frac{\pi}{2} - 1 = x}$

c) $y'' \Rightarrow y' = \dfrac{1}{1-\sin y} = (1-\sin y)^{-1}$

$y'' = -1(1-\sin y)^{-2}(0 - \cos y\, y')$

$= \dfrac{\cos y\left(\frac{1}{1-\sin y}\right)}{(1-\sin y)^2} = \dfrac{\cos y}{1-\sin y} \cdot \dfrac{1}{(1-\sin y)^2} = \dfrac{\cos y}{(1-\sin y)^3}$

$\dfrac{\cos y}{(1-\sin y)^3}$

(8) $y^2 = 4+x$

$2yy' = 1$

$y' = \frac{1}{2y} = \frac{1}{2}$

$\boxed{y = 1}$

$1^2 = 4+x$

$1 = 4+x$

$\boxed{x = -3}$

$\boxed{(-3, 1)}$

$(-4,0)\ (0,2)$

$\frac{2-0}{0+4} = \boxed{\frac{1}{2} = \text{slope} = y'}$

9) $\frac{dy}{dx} = y^2(6-2x) \Rightarrow 6y^2 - 12xy^2$

$y'' = y^2(0-2) + (6-2x)\,2yy'$

$= -2y^2 + 12yy' - 4xyy'$

$= -2y^2 + 12y(6y^2 - 12xy^2) - 4xy(6y^2 - 12xy^2)$

$= -2y^2 + 72y^3 - 144xy^3 - 24xy^3 + 48x^2y^3$

$= 2y^2(-1 + 36y - \underbrace{72xy - 12xy}_{-84xy} + 24x^2y)$

$= 2y^2(24x^2y - 84xy + 36y - 1)$

$2(1/4)^2 * (24*9*(1/4) - 84*3*(1/4) + 36(1/4) - 1)$

$\boxed{= -1/8}$

10) $\frac{dy}{dx}\left[xy^2-x^3y=6\right]$

$= x\cdot 2y\frac{dy}{dx}+y^2(1)-\left[x^3\frac{dy}{dx}+y\cdot 3x^2\right]=0$

$= 2xy\frac{dy}{dx}+y^2-x^3\frac{dy}{dx}-3x^2y=0$

$= 2xy\frac{dy}{dx}-x^3\frac{dy}{dx}=3x^2y-y^2$

$$\frac{dy}{dx}=\frac{3x^2y-y^2}{2xy-x^3}$$

b) $(1,3)$ $\quad 1(y^2)-1^3(y)=6$ $\quad (1,-2)$

$y^2-y-6=0$

$(y-3)(y+2)=0$

$y=3 \quad y=-2$

$\frac{3(1)^2(3)-9}{2(1)(3)-1} = \frac{3\cdot 3-9}{6-1} = \boxed{0}$ slope = 0 $\boxed{y=3}$

$\frac{3(1)^2(-2)-4}{2(1)(-2)-1} \quad \frac{-6-4}{-4-1} = \frac{-10}{-5} = 2$ - slope =

$\boxed{y+2=2(x-1)}$

c) $2xy-x^3=0$ $\quad 0\neq 6 \quad$ DNE

$x(2y-x^2)=0$

$x=0 \quad 2y-x^2=0$

$2y=0$

$y=0$