

Worksheet on Optimization

1. 24 and 8
2. 50 and 25
3. Area = $x(120 - 2x)$
 $x = 30$ ft.
4. Length = $40 - x$
Area = $x(40 - x)$
400 sq. ft.
5. 433.5 sq. m
6. Two squares give 1250 sq. m.
One square gives 2500 sq. m.
7. (a) Circumference = 17.596 cm and
perimeter of square = 22.404 cm
 }- (b) Just a circle with circum. of 40 cm
gives area of 127.324 sq. cm.
- 8. All 4 ft for the circle; none for the square
- 9. Sides of triangle = 1.883 m and
sides of square = 1.087 m
- 10. Radius of circle = 1.120 and
sides of square = 2.240
- 11. 6 in. x 6 in. x 3 in.
- 12. 9 in x 6 in.
- 13. \$330, 3 x 3 x 4m
- 14. $r = 2$ in, $h = 6$ in, \$5.65
- 15. 1.733 hr, 4.5 miles
- 16. \$996,862.70, 6.803mi

WS Optimization

①

$$xy = 192 \quad y = \frac{192}{x}$$

$$x + 3\left(\frac{192}{x}\right) = x + \frac{576}{x}$$

$$f'(x) = 1 - \frac{576}{x^2} = 0$$

$$-\frac{576}{x^2} = -1$$

$$\frac{576}{x^2} = 1$$

$$\frac{192}{x} = 8$$

$$Y = 8$$

②

$$x + 2y = 100 \quad 2y = 100 - x$$

$$xy = \max \quad y = \frac{100-x}{2}$$

$$x\left(\frac{100-x}{2}\right)$$

$$r(x) = 50x - \frac{1}{2}x^2$$

$$f'(x) = 50 - x = 0$$

$$-x = -50$$

$$x = 50$$

$$Y = \frac{50}{2} = 25$$

③

$$2x+y = 120$$

$$y = 120 - 2x$$

$$A = xy$$

$$A = y(120 - 2x)$$

$$= 120x - 2x^2$$

$$A' = 120 - 4x = 0$$

$$\frac{-4x = 120}{x = 30}$$

④

$$P = 2x + 2y + 20$$

$$80 = 2(20) + 2x$$

$$2x = 20$$

$$x = 10$$

$$A' = 2(20) + 2x$$

$$= 2(20) + 2(10) = 60$$

(4)

$$P = 40$$

$$2x + (40 - x) = 40$$

$$2y = 102 - 3x$$

$$A = x(40 - x)$$

$$y = 51 - \frac{3}{2}x$$

$$A = 40x - x^2$$

(5)

$$P = 102 = 3y + 2y$$

$$A = xy$$

$$A = 51x - \frac{3}{2}x^2$$

$$51 - 3x = 0$$

$$-3x = -51$$

$$x = 17$$

$$102 = 51 + 2y$$

$$51 = 2y$$

$$y = \frac{51}{2}$$

$$A = \frac{51}{2} \cdot 17 = \boxed{433.5}$$

(6)

$$7x = 200$$

$$x = 25$$

$$8x = 200$$

$$x = 25$$

$$A = x^2$$

$$A = 25^2 = \boxed{625}$$

$$8x^2 = 200 \Rightarrow x^2 = 25 \Rightarrow \boxed{x = 5}$$

(7)

cont

$$r = \frac{z - 8}{\pi}$$
$$r = \frac{z - 2(0.560)}{\pi}$$
$$r = \frac{z - 2}{\pi}$$

(8)

$$A = \pi r^2$$

$$T.A. = x^2 + \pi r^2$$

$$L = x^2$$

$$T.P. = 4x + 2\pi r = 4$$

$$C = 2\pi r$$

$$2\pi r = 4 - 4x$$

$$\rho = 4x$$

$$r = \frac{2 - 2x}{\pi}$$

$$A(0) = 8^2 + \pi \left(\frac{2}{\pi}\right)^2$$
$$= 64 + 4$$

$$A(0.560) = (0.560)^2 + \pi \left(\frac{2}{\pi}\right)^2$$
$$= 0.3136 + 4$$

$$(2 - 2x)(2 - 2x)$$
$$= 4 - 8x + 4x^2$$

$$A(4) = 4^2 + \pi$$
$$= 16$$

$$A(0) =$$

$$A(0.560) =$$

$$A(4) =$$

$$A = x^2 + \pi \left(\frac{2 - 2x}{\pi}\right)^2$$

$$= x^2 + \frac{4x^2 - 8x + 4}{\pi}$$

$$= \frac{5x^2 - 8x + 4}{\pi}$$

$$A' = \frac{1}{\pi} (2\pi x + 8x - 8) = 0$$

$$2\pi x + 8x - 8 = 0$$

$$2x(\pi + 4) - 8 = 0$$

$$2x = \frac{8}{\pi + 4}$$

$$x = \frac{4}{\pi + 4} \approx 0.560$$

1. On Optimization

$$A = \pi r^2$$

$$\text{T.A.} = x^2 + \pi r^2$$

$$A = x^2$$

$$\text{T.A.} = 4x + 2\pi r = 40$$

$$C = 2\pi r$$

$$P = 4x$$

$$2\pi r = 40 - 4x$$

$$r = \frac{40 - 4x}{2\pi} = \frac{20 - 2x}{\pi}$$

$$\text{T.A.} = x^2 + \pi \left(\frac{20 - 2x}{\pi} \right)^2$$

$$(20 - 2x) (20 - 2x)$$

$$= x^2 + \frac{400 - 80x + 4x^2}{\pi}$$

$$= \frac{\pi x^2 + 4x^2 - 80x + 400}{\pi}$$

$$\text{T.A.'} = \frac{1}{\pi} (2\pi x + 8x - 80) = 0$$

$$2\pi x + 8x = 80$$

$$2x(\pi + 4) = 80$$

$$2x = \frac{80}{\pi + 4}$$

$$x = \frac{40}{\pi + 4} \approx 5.60099$$

$$P = 4(5.60099) = 22.403$$

$$40 = 2\pi r$$

$$r = 6.3662$$

$$\pi r^2 = 127.324$$

$$C = 2\pi r = 40 - 4x$$

$$= 2\pi r = 40 - 4(5.60099)$$

$$C = 2\pi r = 17.576$$

$$r = 2.8005$$

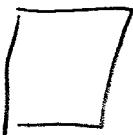
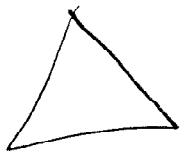
$$10^2 = 100$$

Circle with

Area ≈ 127.324

at radius \approx

6.3662



$$A = x^2$$

$$A = k_2 y h$$

$$c = 3y$$

$$3y + 4x = 10$$

$$T.A. = x^2 + k_2 Y \frac{\sqrt{3}y^2}{2} \quad P = 4x$$

$$T.P. = 3y + 4x = 10$$

$$\begin{aligned} (\frac{1}{2}Y)^2 + h^2 &= r^2 \\ Y^2 + h^2 &= r^2 \\ h^2 &= r^2 - Y^2 = \frac{r^2}{4} - \frac{3y^2}{4} \\ h &= \sqrt{\frac{r^2 - 3y^2}{4}} \\ h &= \frac{\sqrt{r^2 - 3y^2}}{2} \end{aligned}$$